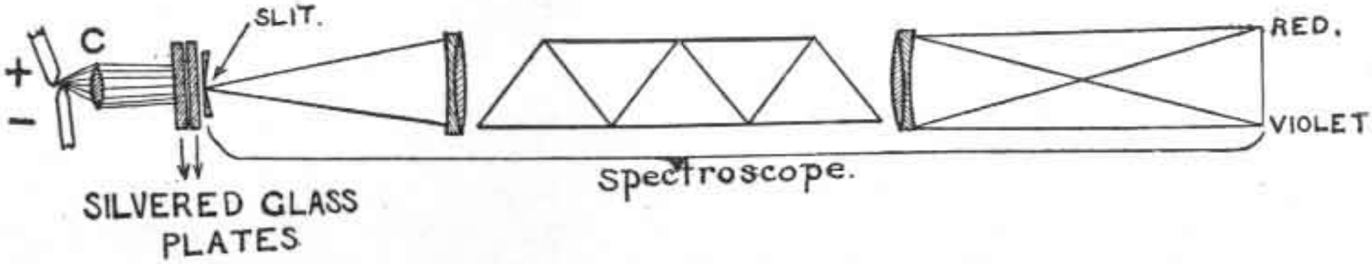
Edser, E & Butler, C.P., "Simple Method of Reducing Prismatic Spectra", Philos. Mag. 46, pp. 207-216 (1898)

Fig. 2.

Diagrammatic Plan of Apparatus.



Fabry-Perot interferometer not Michelson

Fringes are non-sinusoidal due to multiple reflections; angle-dependent delay value restricts use to small delays and point sources

Edser, E & Butler, C.P. year=1898 vol =46 page = 207

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AND

JOURNAL OF SCIENCE.

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GEORGE FRANCIS FITZGERALD, M.A. Sc.D. F.R.S.

AND

WILLIAM FRANCIS, Ph.D. F.L.S. F.R.A.S. F.C.S.

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XV. *A Simple Method of Reducing Prismatic Spectra.* *By* Edwin Edser, *A.R.C.S., and* C. P. Butler, *A.R.C.S.**

IN order to determine, from spectroscopic measurements, the wave-lengths corresponding to the bright lines in a prismatic spectrum of a metal or gas, one or other of the following methods is generally used. From preliminary measurements made of the deviation corresponding to a number of known lines in the solar spectrum, or the line spectrum of some metal or gas, a curve is drawn giving the relation between deviation and wave-length. Owing to the necessity of determining a very large number of points on this curve in order to render its form trustworthy, this operation is a very tedious one, and to an observer insufficiently acquainted with the reference spectrum involves great difficulty and uncertainty. This curve, however, having been drawn, the wave-length of any line in another spectrum obtained with the same spectrometer (no alteration of the adjustments having been made) can be immediately determined from a measurement of its deviation. On the other hand, where photographs of spectra are employed the most usual practice at present is to photograph a reference solar-spectrum alongside the one under examination. To an observer of sufficient experience it is possible to identify any of the numerous Fraunhofer lines with the corresponding lines in a Rowland's standard map; and thus the wave-length of any line in the unknown spectrum may be determined by inspection. In spite of the perfection attainable by the above methods when employed by a trained observer, it has appeared to us that a simpler one, capable of giving accurate results in the hands of an experimenter without special experience in spectroscopy, might often be found of some value. The production of interference-bands in a continuous spectrum seemed capable of furnishing a reference spectrum which could be advantageously employed for this purpose, most of the difficulties incident to the above-mentioned methods being entirely eliminated. We have, therefore, devoted some time to the examination of various methods by which such interference-bands might be produced, with the object of selecting the simplest, and determining the degree of accuracy finally attainable by its employment. The results of our work in this direction we beg to lay before the Society this evening.

If the image of a system of rectilinear interference-fringes be formed in the plane of, and parallel to, the collimator slit of a spectrometer so that only a small part of the breadth of

* Communicated by the Physical Society: read May 27, 1898.

one band falls on the slit, the resulting spectrum will be crossed by vertical black bands, varying in number and breadth with the order of the interference-band from which the spectrum is derived*. If the interference-fringes are displaced across the slit the black bands in the spectrum will become finer and more numerous as the central interference-fringe recedes from the slit. Even when the coloured fringes have become invisible at the position of the slit owing to the high relative retardation of the interfering pencils, the bands in the spectrum remain quite distinct, becoming indistinguishable only when so fine that the resolving and dispersive powers of the spectroscope are insufficient to separate them.

In our earlier experiments we focussed the image of an air-film, contained between two plane and parallel glass surfaces, on the slit of the spectrometer †. The image should be obtained by means of light reflected from the film, the spectrum bands obtained when transmitted light is used being very faint. This method, which is theoretically the most perfect, has the disadvantage that a somewhat careful adjustment is necessary in order to insure good results. We have therefore sought for some simpler method. It is unnecessary here to detail the various methods which we have successively tried; it will suffice to describe the arrangement ultimately adopted as being the simplest, whilst complying sufficiently closely with the ideal conditions to insure trustworthy results.

Let us suppose that a transparent parallel-sided film of thickness d is placed immediately against the slit of the spectroscope and illumined with white light. Owing to the interference of the ray directly transmitted and that twice internally reflected within the film there will be bright bands in the spectrum separated by darker intervals, the wave-lengths $\lambda_0, \lambda_1, \lambda_2, \ldots \lambda_r, \ldots \lambda_m$ corresponding to the bright bands being given by the equations

$$2\mu d = n\lambda_0 = (n+1)\lambda_1 = (n+2)\lambda_2 = \ldots = (n+r)\lambda_r = (n+m)\lambda_m,$$

where μ is the refractive index (supposed independent of the wave-length) of the substance of the film, whilst n may be any integral number.

If λ_0 and λ_m are known n can be determined from the equation

$$n = \frac{m\lambda_m}{\lambda_0 - \lambda_m}, \quad \ldots \ldots \quad (1)$$

* Fizeau and Foucault, *Ann. de Chim. et de Phys.* 3rd series, tom. xxvi. p. 138 (1849); *Comptes Rendus*, Nov. 24, 1845.

† An air-film has been used somewhat similarly by Rubens in order to calibrate a prism for infra-red light. Wied. *Ann.* vol. xlv. (1892) p. 238.

where the interference-band at λ_m is the mth from that at λ_0; λ_m being towards the violet, and λ_0 towards the red end of the spectrum.

The wave-length λ_r corresponding to any other interference band (the rth from that at λ_0) is now immediately given by

$$\lambda_r = \frac{n\lambda_0}{n+r}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

It is therefore possible, by means of a series of interference-bands produced in the spectrum in the above manner, to calculate the wave-length corresponding to any part of the spectrum, having given any two lines of known wave-lengths sufficiently remote from each other.

Of course it is impossible to obtain a solid film of any substance whose dispersion is sufficiently small to render the above reasoning even approximately correct. Recourse must then be had to an air-film between two transparent plates. Since the film can now no longer be placed immediately against the collimator slit, some indefiniteness of the interference-bands will result; but if the plate next to the slit is not more than 3 millim. in thickness no trouble will arise from this cause, at any rate with a spectrometer whose collimator tube is more than a foot in length.

A very considerable improvement in the interference-bands thus produced may be effected by partially silvering the two surfaces enclosing the air-film. In the first place the contrast between the bright and dark bands is considerably enhanced; indeed, if both surfaces be silvered so as to reflect about 75 per cent. of the incident light, the dark spectrum bands become almost black. The thicker the silver is the greater will be the contrast, the only limit being prescribed by the diminution of the total light transmitted.

Another important advantage gained by silvering the surfaces is the much sharper definition of the resulting bands. Messieurs C. Fabry and A. Perot* have pointed out that when monochromatic light is transmitted through a film enclosed between two plane and approximately parallel silvered surfaces, the resulting interference-bands present the appearance of sharp well-defined bright lines separated by broad black intervals. The explanation of this interesting phenomenon is quite simple. Let the real part of $\epsilon^{ip(x-Vt)}$ be the equation of the incident wave, whilst a and b are the respective coefficients of reflexion and transmission at the silver surface. Since Wiener has shown that the phase change for light reflected normally at a silver surface in air

* *Ann. Chim. Phys.* xii. pp. 459–501 (1897)

is very approximately equal to half a wave, a may be taken as wholly real.

Consequently, if the thickness of the air-film is d, the resultant transmitted beam on emergence will be given by the real part of the sum of the infinite series

$$b^2\epsilon^{\iota p(x-\mathrm{V}t+d)}+b^2a^2\epsilon^{\iota p(x-\mathrm{V}t+3d)}+b^2a^4\epsilon^{\iota p(x-\mathrm{V}t+5d)}+\text{\&c.}$$

$$=b^2\epsilon^{\iota p(x-\mathrm{V}t+d)}\{1+a^2\epsilon^{2\iota pd}+a^4\epsilon^{4\iota pd}+\text{\&c.}\}$$

$$=b^2\epsilon^{\iota p(x-\mathrm{V}t+d)}\cdot\frac{1}{1-a^2\epsilon^{2\iota pd}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Substituting $\cos 2pd+\iota\sin 2pd$ for $\epsilon^{2\iota pd}$, and rationalizing the denominator of (3), we find the transmitted wave to be equal to the real part of

$$\mathrm{A}\epsilon^{\iota p(x-\mathrm{V}t+d+e)},$$

where

$$\mathrm{A}^2=\frac{b^2}{1+a^4-2a^2\cos 2pd}, \quad . \quad . \quad . \quad . \quad (4)$$

and

$$\tan pe=\frac{\sin 2pd}{1-a^2\cos 2pd}.$$

Now A^2 is proportional to the intensity of the transmitted light; hence as d is varied the intensity will vary from $\mathrm{I}_{(\max.)}=\frac{b^2}{(1-a^2)^2}$ to $\mathrm{I}_{(\min.)}=\frac{b^2}{(1+a^2)^2}$. Taking Michelson's expression for the visibility of interference bands, viz., $\frac{\mathrm{I}_{(\max.)}-\mathrm{I}_{(\min.)}}{\mathrm{I}_{(\max.)}+\mathrm{I}_{(\min.)}}$, we find that this becomes equal to $\frac{2a^2}{1+a^4}$, which will have a maximum value when $a=1$. Thus the visibility of the bands will increase with the reflecting power, and therefore with the thickness of the silver.

Also, from 4, we obtain

$$\frac{d\mathrm{I}}{dd}=c\frac{-4pa^2b^2\sin 2pd}{(1+a^4-2a^2\cos 2pd)^2}.$$

Consequently I varies much more rapidly as d is increased when $2pd$ is nearly equal to $2n\pi$ than it does when $2pd$ is nearly equal to $(2n+1)\pi$. The bright bands will therefore be very narrow and sharply defined, separated by broad intervals very nearly black *.

* A similar result has been noticed in connexion with the interference of electrical waves. See "Electrical Interference Phenomena, somewhat analogous to Newton's Rings, but exhibited by waves passing along wires of which a part differs from the rest." By E. H. Barton, D.Sc., Proc. Roy. Soc. vol. liv. p. 85.

It may be useful here to describe in detail the exact method of procedure finally adopted. It has not been found necessary to use optically worked glass; good ordinary plate-glass gives perfect results. Sextant glasses have been recommended to us for this purpose. It is well to select two plates having the most suitable surfaces. This can be done by placing one plate on another, the two adjacent surfaces having previously been cleaned with cotton-wool, and viewing the air-film between them by reflected light from a sodium flame. The bands formed when the plates have been gently pressed together should be nearly straight and each one at least 2 millim. or 3 millim. in breadth.

The two selected surfaces should now be silvered somewhat heavily. The milk-sugar process*, in which the silver is deposited very slowly, has been found to give good results.

A simple mechanical arrangement for adjusting the two silvered surfaces for parallelism, the distance between them being also capable of adjustment, could easily be designed. We have found, however, that if a little soft wax be placed round the edges of the plates a perfect adjustment can be obtained by simply pressing the plates together with the fingers. The photograph accompanying this paper was obtained using this arrangement. To adjust for parallelism, view a spot of light, or the filament of an incandescent electric lamp, through the silvered surfaces. A long train of images, due to multiple reflexions, will generally be visible. These images having been brought into coincidence, interference-bands will generally be seen on viewing a sodium flame through the silvered surfaces. These are adjusted, by pressure applied to the glass plates, to be as broad as possible. When the adjustment is nearly completed there is often some difficulty in seeing the bands, due to the fact that for a parallel air-film viewed normally the interference-bands are formed at an infinite distance in front of the film †. At this stage the bands should be viewed from as great a distance as possible. The perfection of the results finally obtained will depend greatly on the accuracy with which this adjustment is performed.

If the collimator slit of the spectrometer be now illumined by a slightly convergent beam from an arc-lamp, and the plates be placed in front of the slit, and as near to it as

* For the exact process employed by us see 'Nature,' Sept. 23, 1897, "On the Phase-change of Light when Reflected from a Silver Surface," by Edwin Edser and H. Stansfield.

† A. A. Michelson on "Interference Phenomena in new form of Refractometer," Phil. Mag., April 1882.

possible, the spectrum will be found to consist of bright lines separated by almost black intervals. The best results will of course be obtained when the plates are in such a position that the slit is parallel to the direction of the interference-bands seen with sodium light. The closeness of the bands will depend on the thickness of the air-film between the silvered surfaces. For photographic purposes we have adopted the plan of covering either the upper or lower half of the slit with a piece of black paper stuck on with soft red wax before placing the plates in position. The necessary exposure will vary from about half a minute to three minutes (using Edwards's snap-shot isochromatic plates) according to the nature of the spectrometer employed. It is well to introduce a little common salt into the arc while this exposure is being made, as thus the D lines, as well as the H and K lines, will be superimposed on the bands. Another piece of black paper having been placed so as just to cover the exposed half of the slit, the first piece is removed, and the spectrum which it is wished to examine is photographed. Fig. 1 is a specimen of an iron spectrum together with a reference interference-spectrum obtained in this way. It will be noticed that the interference-bands in the violet part of the spectrum are slightly inclined to the vertical. It is easy to adjust the glass plates so that this is not the case, but we have selected this photograph in order to point out that if readings are taken on the line of junction of the two spectra no error will result from such a want of adjustment.

Starting from the red end of the spectrum every fifth and tenth band can be marked and the whole numbered. The following exhibits the procedure when the wave-lengths of only a few lines are required.

One or other of the D lines and the H or the K line will generally be found to form the best datum lines. In the present case the following datum lines were used :—

Scale No. 90·2. Wave-length 5328·5 (λ_0)
" 402·3. " 3968·6 (λ_m) (H).

Then according to equation (1)

$$n=\frac{m\lambda_m}{\lambda_0-\lambda_m},$$

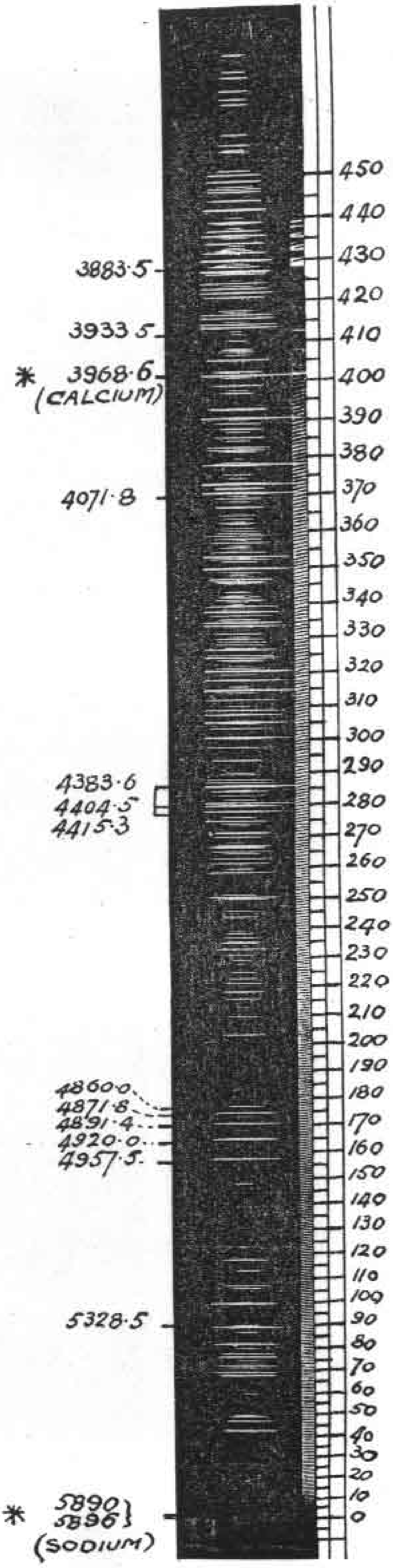
$$m=402{\cdot}3-90{\cdot}2=312{\cdot}1,$$

$$\lambda_0-\lambda_m=1359{\cdot}9\,;$$

hence

$$n=910{\cdot}8.$$

Fig. 1.



COMPARISON SCALE FOR REDUCTION OF SPECTRA.

To find the wave-length of the line whose scale-number $=371\cdot2$:

$$r=371\cdot2-90\cdot2=281,$$

$$\lambda_r=\frac{n\lambda_0}{n+r}=\frac{910\cdot8\times5328\cdot5}{910\cdot8+281}$$

$$=4072\cdot2.$$

The true value of this wave-length is 4071·8, giving an error of +·4 tenth-metres.

The following Table shows the calculated and true values for a number of lines in the above spectrum. It is given in order to indicate the degree of accuracy attainable. It is worth notice that these results were all obtained without the use of a travelling microscope, or in fact any auxiliary appliance other than an ordinary pocket-lens. With the latter it is easy to estimate the position of a line relatively to the interference-scale to within one-tenth of a band. Further, the interference-scale in the present instance was purposely made rather coarse so as to admit of reproduction. With a finer scale a greater degree of accuracy might be attained.

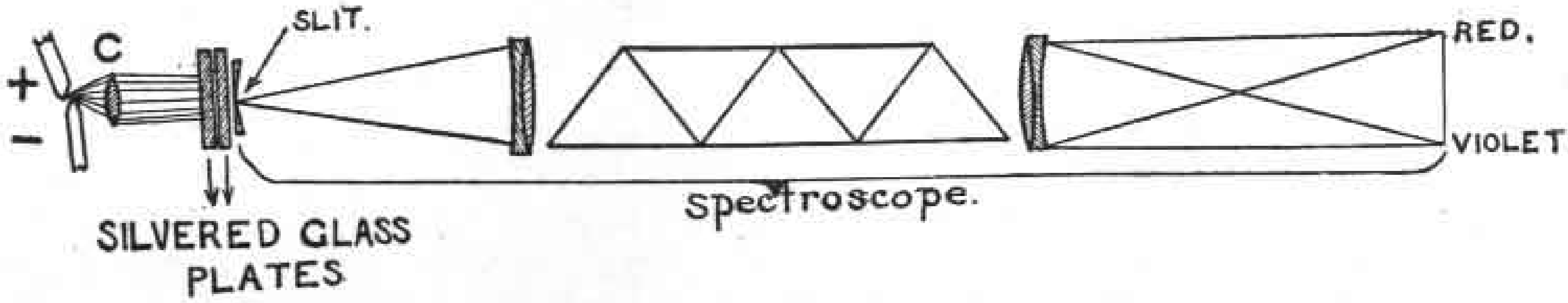
Scale No.	Wave-length (calculated).	True Wave-length.	Error.
			tenth-metres.
371·2	4072·2	4071·8	+·4
286·5	4383·7	4383·6	+·1
281·1	4405·2	4404·8	+·4
278·6	4415·2	4415·3	−·1
354	4131·8	4132·2	+·4
413·1	3933·9	3933·5 (K)	+·4

When it is required to determine the wave-length corresponding to a great number of spectral lines a graphical method may be employed. If we write $\frac{1}{\lambda}=\mathrm{L}=$ the frequency of the light vibrations, we obtain the simple relation $\frac{n+r}{\mathrm{L}}=$ constant, or $\mathrm{L}=\mathrm{K}(r+n)$, *i. e.* the relation between r and L may be expressed by a straight line.

Plotting the frequencies vertically, and r horizontally, we obtain fig. 3. It is only necessary to mark off the scale-divisions, starting from zero, along the horizontal axis, and to mark off vertically above their respective scale-divisions the frequencies of the two standard lines, joining the extre-

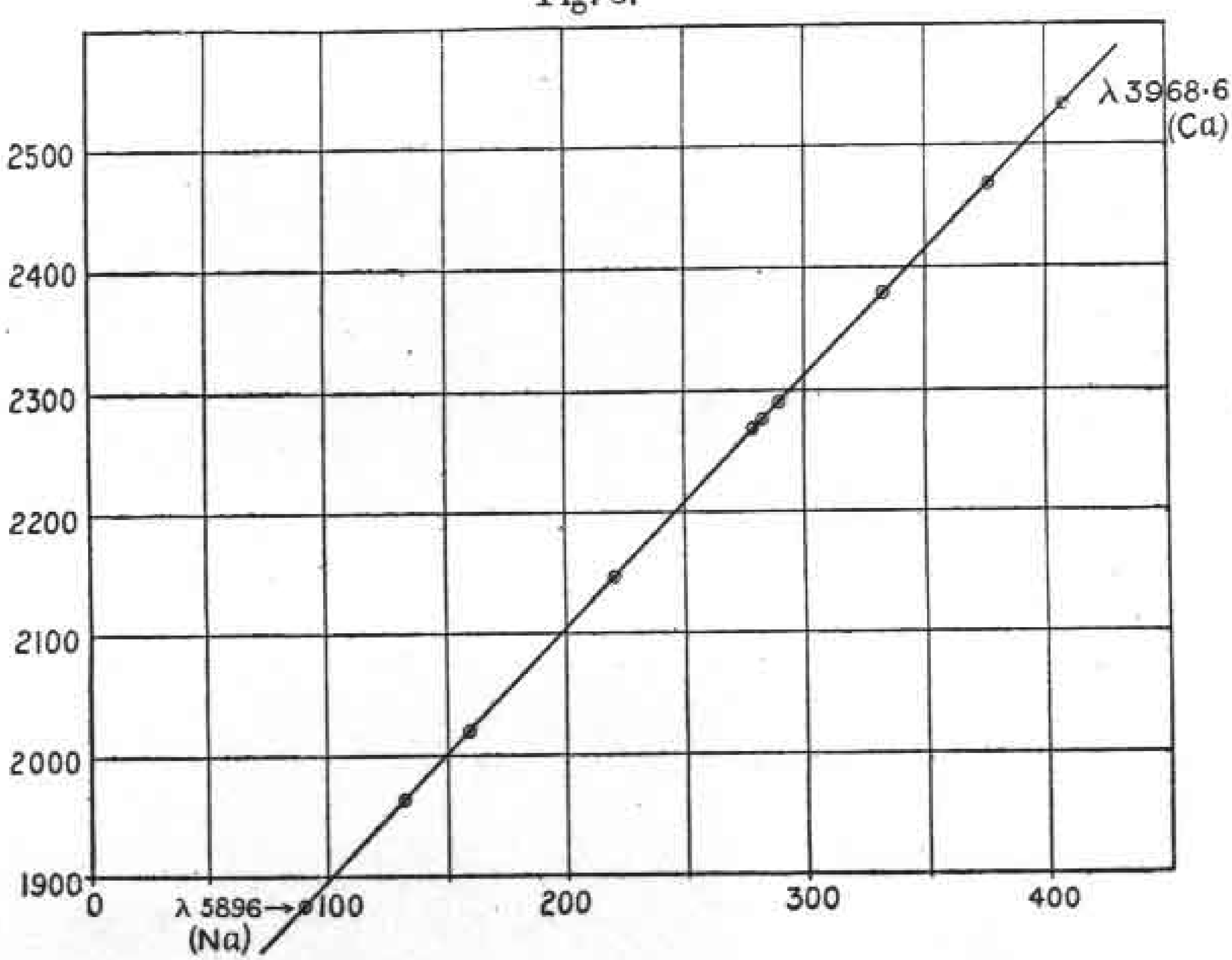
Fig. 2.

Diagrammatic Plan of Apparatus.



mities of the latter by a straight line. The frequency corresponding to any given scale-number is read off directly. The accuracy attainable by this method, in which no calculations whatever are involved, is similar to that obtained by the method previously described.

Fig. 3.



It is, of course, unnecessary to take a separate photograph of the reference interference-scale for every spectrum to be examined. If the D lines are superposed on the original interference-scale, and occur also in every succeeding spectral photograph obtained, the reference scale can be photographed once for all, provided the adjustments of the spectrometer remain unaltered. The photographic scale can be placed with its film in contact with that of the photograph bearing the unknown spectrum, and the D lines having been brought into coincidence the procedure indicated above may be proceeded with.

For eye-observations the most convenient arrangement would be to place a small plate of optically worked glass between the reference prism, generally provided with a spectrometer, and the slit, a simple arrangement serving to adjust the adjacent surfaces (which should be silvered) for parallelism.

It will be seen that the phase-changes produced by the silver do not introduce any serious errors into the final results. Wiener* has shown that for light reflected from a silver film of sufficient thickness the phase-change is very nearly independent of the wave-length. To further test this a streak of silver was rubbed off the glass plate which is placed next to the collimator slit, and a photograph of the spectral bands obtained. The displacement of the bands, where the light had been reflected from the silver, relatively to the bands formed where the light had been reflected from the clear glass, was practically constant for the whole length of the spectrum.

In conclusion, we think that it may be claimed that by means of this application of a well-known principle to spectroscopy, the difficulties incident to the reduction of prismatic spectra in terms of wave-lengths or frequencies are greatly reduced, the whole process as above described requiring no special experience in the experimenter.

The experimental work incidental to this investigation has been performed partly at the Davy-Faraday Laboratory, Royal Institution, and partly at the Royal College of Science, South Kensington. For the facilities afforded us individually at these institutions our joint thanks are due.

XVI. *Note on the Measurement of Colour and the Determination of White Light.* *By* T. E. DOUBT, *University of Washington, Seattle, Washington* †.

IN his paper on the "Theory of Compound Colour" Maxwell has given colour-equations to represent white light. From these equations, by eliminating the quantity W between any two equations, the relation between any three colours may be obtained. In making his determinations he used a white diffusing-screen illuminated by direct sunlight. The quality of the light that is reflected by a screen depends somewhat

* O. Wiener, Wien. *Ann.* xxxi. p. 629 (1887).

† Communicated by the Author.