ABSOLUTE ASTRONOMICAL ACCELEROMETRY

PIERRE CONNES

Service d'Aéronomie du C.N.R.S., Verrières-le-Buisson, France

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Abstract. Two distinct but fully compatible novel concepts are proposed here for solar/stellar velocity measurements. The first is that of absolute accelerometry proper. This involves two simultaneously operating servo-control loops. First, a variable path-difference Fabry–Perot interferometer is adjusted so that its bandpasses track the fluctuations of either a single spectral line (in the solar case, leading to the solar accelerometer), or of all lines simultaneously (stellar accelerometer). The second loop involves a tunable laser tracking one of the FP bandpasses. The net overall result is that a laser line tracks the stellar/solar ones: the problem of measuring Doppler-shift changes has been transferred from the incoherent to the coherent optics domain. One then measures the beat frequency generated by mixing the tunable laser beam with that of stabilized laser. Only velocity changes are accessible; the devices are true accelerometers, but absolute ones. All instrumental or spectral characteristics drop out; no calibration of any kind is required; hence, one may hope for an unusually low level of systematic errors.

The second concept is that of optimum measurement of Doppler shifts as far as photon count limitations are concerned. A simple but so far never performed calculation leads to the fundamental RMS velocity error corresponding to a given spectral profile and photon count. One next shows that a dispersive spectrometer with an image detector may closely approach that limit provided direct access to a computer is available, and the signal is treated by a specific algorithm. This treatment being precisely the one used in the stellar accelerometer, our device is seen as the first proposed one approaching fundamental limits in this field; however, standard radial velocity measurements (not involving accelerometry) should also benefit from our proposal. A full calculation shows that a velocity error reduction of the order of 30 is within reach relative to the most efficient so far available device, i.e., CORAVEL. For faint objects, detector noise must be added, but the treatment remains demonstrably optimum.

The two principal fields of application for absolute accelerometry are celestial seismology (a seismometer is nothing but an accelerometer), and the search for extra-solar planetary systems. In both cases a large number of objects will be accessible with a small telescope. One may also look for solar system accelerations (relative to some system of reference stars) due to any cause whatsoever: for instance a faint solar companion, or even gravitational waves.

> "When Infinite Systems succeed one another through an Infinite Space, and none is either inward or outward; may not all the Systems be situated in an accurate Poise; and, because equally attracted on all sides, remain fixed and unmoved? But to this we reply; that unless the very mathematical Center of Gravity of every System be placed and fixed in the very mathematical Center of the Attractive Power of all the rest; they cannot be evenly attracted on all sides, but must preponderate some way or other."
>
> RICHARD BENTLEY (1926)

1. Introduction

The problem of detecting and accurately measuring solar or stellar accelerations arises in two different astrophysical contexts, namely:

(1) *Stellar seismology.* As far as our Sun is concerned, this is a rapidly developing field in which striking results have been collected over the last few years. By measuring surface radial motions from wavelength fluctuations of selected lines, many proper

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modes of solar oscillations have been found. However, alien starquakes are still seen mostly through the mind's eye, for lack of a suitable detecting technique. Still the subject is most appealing; we would if we could, but we are not able.

(2) *Search for extrasolar planetary systems*. This is a quest that has yet to produce one discovery. Out of the three main possible techniques, direct, astrometric and spectroscopic, the third (and only one to be discussed here) is possibly the least fraught with technical difficulties; it involves measuring the radial component (as seen from Earth) of the putative planetary system central star induced periodic motion around the common center of gravity.

These two problems are totally unconnected scientifically. They are brought together here merely because the required technology is essentially the same. In both cases the basic tool has been delivered by Christian Doppler and Hippolyte Fizeau and is one of the most commonly wielded in astronomy. However, both stand apart from regular radial velocity work because of one common fact: the kinematical quantity to be measured is not velocity but acceleration, an essential difference perhaps not clearly perceived, or at least not put to good use so far. The point is stressed here not through mere pedantry: there are two important consequences. First, most astrophysical limitations to the accurate determination of radial velocity do not apply. Because of turbulent motions in the atmosphere of any star, plus various effects, we cannot hope (even if the tools were available) to define by any optical means whatsoever the radial velocity of the star center of gravity with an error less than perhaps 100 m s^{-1} (Dravins, 1975). Even in the case of the Sun, the detection of the gravitational shift, which is equivalent to a Doppler shift of 600 m s^{-1}, is still somewhat questionable. However, on the very same Sun, we are already detecting seismic tremors in the spectrum of which the weakest reliably isolated lines correspond to an oscillatory radial velocity with an amplitude less than 10 cm s^{-1} (Grec *et al.*, 1980), and there is a general agreement that distinctly higher accuracy is both desirable and feasible.

The second consequence is that all those widely available spectrometers which have been developed for radial velocity work, even of the most accurate kind, are not *specifically* suited to the job. Even in the case of the Sun for which, as stressed above, we do have results and quite remarkable ones, these have been collected using either standard solar magnetographs or alkaline vapour cells, both powerful tools indeed but ones which had been developped long before sunquakes had even been thought of. In short we do have a budding solar seismology; we do not yet have a solar seismometer. Which is, no doubt, vastly better than the reverse. Still it is now right and proper to attack the problem in earnest: the happy epoch of early unsophistication is fast passing away. Elaborate (and expensive) projects are already being considered, particularly in space – to wit, the DISCO proposal, presently studied by ESA, whose main program will be solar seismology.

Having no results at all to show, extrasolar planetary search is not so far such a fashionable field; still, in a similar way, the used or proposed tools are chiefly some initially designed for other fields or, at most, conceptually minor adaptations of existing ones. None has been developed to the point of having fully demonstrated on any star adequate accuracy, i.e., the ability to detect a 12 years period, 13 m s^{-1} amplitude Doppler shift (these figures correspond to the 'standard' case of a Jupiter-like planet orbiting a Sun-like star). Admittedly, the uncertainties in this case are not purely instrumental and there may well be astrophysical limitations, though how large is presently unclear. We certainty cannot hope to measure the *absolute* radial velocity of any star to 13 m s^{-1}; but even when long period oscillations with comparable amplitude are considered, it is conceivable that stellar phenomena might masquerade as periodic shift of the CG. Even for our Sun, the obvious testing ground, the answer is not known; no fully suitable instrument is available, and no dedicated (which means among other things, long term) program of study has yet been implemented.

Nevertheless at least one prediction appears fully safe. A stellar accelerometer with the proven ability to detect velocity changes of the order of 1 m s^{-1} or better would unquestionably produce worthwhile results. In the very short time range (minutes or hours), star quakes would no doubt be detected, possibly for a wide range of spectral types; and such seismology seems presently our best bet for understanding stellar interiors. In the longer time range (days to months to years) a large number of dark companions of various sizes would just as certainly be found; and if quasi-periodic phenomena in the stellar atmospheres did ultimately hinder the detection of planetary sized companions, then these phenomena in themselves might open up a new field of study.

Lastly we may play with the idea that such a device might help to solve one of the very first questions asked after the concept of universal gravitation became clear: does the Sun accelerate relative to other stars? For Richard Bentley, in his friendly discussion with Isaac Newton, the absence of any departure from an obviously unstable universal equilibrium was a straightforward proof of the active interference of God within Nature (Hoskin, 1982). It is much to be regretted that the consequences today would not be held as quite so sweeping; otherwise, and keeping to the purely human viewpoint, financing of the whole scheme might be powerfully helped.

What are the available tools? No comprehensive review is intended here; still these should be briefly recalled*. So far our twin problems have been perceived as distinct, and attacked with different instruments. Let us start with seismology. The first tool is the Babcock solar magnetograph used as a velocity-meter, i.e., on a non-magnetic line. This has led for instance to detailed velocity fields, and to the beautiful $k - \omega$ diagrams; however, in the present work (the ultimate aim being study of stars) only *integrated light* solar observations will be discussed or proposed. Since a magnetograph is essentially a large grating spectrograph, it does not exhibit sufficient intrinsic stability to provide a long term velocity reference. Still, results approximating integrated velocities are obtained by a technique first used by Severny *et al.* (1978): the wavelength of light from a wide central region of the solar disk is compared with that of the rim. One sees, however, that velocities obtained in this way cannot be independent of guiding errors,

* A novel and interesting device, from which we do not yet have results, is the Fourier tachometer described by Brown (1980) and Evans (1980). Because of the wide acceptance, it appears particularly well suited to the study of solar velocity fields.

unlike true integrated light ones. The same method has been used at Stanford (Dittmer, 1977). Results include the detection and study over many years of the difficult and somewhat controversial 160 min solar oscillation.

An altogether different approach involves alkaline vapour resonance cells. Many remarkable results, in particular the highly detailed fine structure of the 5 min oscillation have been collected in this way; they are largely responsible for the present fascination with solar seismology. The two instruments have been described by Brookes *et al.* (1978) and Grec *et al.* (1976). The question we would like to ask here is: What are the ultimate limitations of these devices? A full answer is not possible at present; let us at least present the problem. We may conceptually distinguish three sorts of difficulties. The first are relevant to ground based observations: daily interruptions, and atmospheric seeing and transparency effects. These have been analysed by ourselves (Connes and Connes, 1982; Connes, 1984; and also Appourchaux, 1984), the conclusion being that no serious difficulty seems so far involved. Second, we have light collection effects: guiding errors and imperfect integration over the solar disk. Lastly come the sodium or potassium cells themselves for which we may define an efficiency, leading to a photon count velocity limit, plus various systematic errors. In actual practice, unraveling the three respective contributions to the overall error budget is not simple. For instance, both resonance spectrometers at first exhibited large sensitivity to solar image position and azimuth, while there is no fundamental reason why things should be so; however, both have obviously improved since the early days; a full rediscussion of their performance, and potential, by the involved specialists would be helpful indeed. We have presented a tentative one by Connes (1984).

One of their main characteristics must, however, be stressed here in order to make the present proposal understandable. Resonance cells (and similarly atomic jets) provide an *absolute zero-velocity reference* since hopefully identical solar and terrestrial atoms are compared in the most direct possible manner. Consequently, such devices remain unbeatable for the detection of the solar gravitational shift; clearly, in this case, the limitations are astrophysical, not instrumental. However, when *velocity fluctuations* are studied, the system must be *calibrated* from known velocity changes, happily provided by Earth motion. At this point the operation does not differ from that performed on magnetograph data: during each daily run, the general slow trend in the measured velocity is subtracted out, and the small residual ripples are attributed to sunquakes. The procedure is largely responsible for the greater difficulty involved in unraveling the 160 min oscillation, and all gravity modes, compared to the 5 min ones.

There are so far few attempts, and fewer clear results still, when stellar oscillations are considered. Fossat *et al.* (1984) have used a sodium cell in absorption (because photon-using efficiency is much higher than for resonance cells); they present an important result, the first positive evidence for p-modes oscillations in a solar type star. However, this was α Cen ($m_V = -0.1$) and two nights of integration were required with a 3.5 m telescope; this illustrates well the present difficulty of stellar seismology. Traub *et al.* (1978) have worked with a Pepsios multiple Fabry–Perot spectrometer whose bandpass was simply set on a line flank; as in the previous case, only one stellar line could be used. Then, Smith (1982, 1983) has used a large Coudé spectrograph equipped with a linear Reticon detector, and has measured wavelength fluctuations of 7 Arcturus lines; thus, in principle sensitivity must have been somewhat higher than in the other two attempts. Furthermore, employing a technique whose systematic use in this context was first proposed by Griffin (1972), these lines were measured relative to a set of telluric O_2 lines; this eliminates most of the usual grating spectrograph failings, but introduces changing winds into the experiment*.

Extrasolar planetary detection is a very long term proposition, and has only been attempted recently; so far, we do not have results from a full time search, but merely a few isolated observations. Still, these are of great interest since they illustrate feasible accuracy. On one side, we have the above-mentioned work by Smith which, as stressed by the author, would in principle be applicable to planetary search. A similar line of approach is followed by Campbell (1983), who uses for calibration an HF cell; this is potentially susceptible of greater stability. The actual results of these various attempts will be summarized here on our Figure 10. On the other side, we have with the Fabry–Perot radial velocity spectrometer described by Serkowski (1976, 1979) and MacMillan (1982), a complete system built up specifically for planetary search, and presently in the testing stage. Let us reduce it to essentials: the device uses the standard configuration of a FP etalon in series with a crossed dispersion grating-echelle spectrograph; the detector is a two-dimensional CCD on CID array. The system could be used as it stands for any high-resolution spectroscopic work; it has been, however, optimized for precision radial velocities. Only one essential feature must be stressed at this point: any device making use of a FP etalon in the beam from a broad-band source has inherently low efficiency for our problem, of the order of the inverse of the finesse or worse. This point is already made clear by the greater efficiency demonstrated by the CORAVEL system (to be discussed presently), compared to the one computed by MacMillan (1982). The advantage of the FP lies in the hope for much lower level of systematic orders.

Which directions should we look into? The first is that indicated by the 'Radial Velocity Photometer' proposed and discussed by Fellgett (1955) and demonstrated by Griffin (1967). The latest and clearly most efficient version is CORAVEL, built by Baranne *et al.* (1979); a particularly thorough and useful analysis by Poncet (1978) is available. Here we have a completely original device specifically designed for radial velocity work. The principle is now sufficiently familiar not to require a description, and only the main features need be discussed here. The first is a remarkable efficiency coupled with simplicity of operation: one does not have to go through accurate positions for individual lines. The efficiency is due to the large number of lines used, and used in a nearly optimal manner. No scanning of the spectrum is required as with FP interferometers: all spectral elements are *simultaneously* available. Only one has to scan

* Some of the most recent results in the search for stellar oscillations have been presented at the February 1984 Meudon Conference on 'Space Research Prospects in Stellar Activity and Variability'; the proceedings have been edited by A. Mangeney and F. Praderie. Being often referred to here, they will be designated as *Meudon Conference 1984*

the mask-spectrum cross correlation function; the range is very narrow, especially if the star velocity is approximately known in advance.

On the other hand, if we forget for a moment about the photon flux, and consider systematic errors, we find the instrument exhibiting all the usual grating spectrometer errors. Resulting performance (for CORAVEL) is again plotted on our Figure 10, which separates the two phenomena. For bright stars, we get a constant instrumental error of about 120 m s^{-1} (horizontal line), which is not remakably low: large Coudé spectrographs do much better, but CORAVEL had been designed as a compact Cassegrain-focus device for small telescopes. For faint stars, one reaches a photon-count limited performance (slope 1 straight line) with 400 m s^{-1} error at $m_V = 15$ (1 m telescope and 1 hr observing).

It is essential to keep in mind that this photon noise-limited error is a *fundamental* result, while the instrumental error is an *accidental* one: no improvements of standard Fabry–Perot devices will enable them to reach correlation spectrometer performance, but the reverse may well be true. Consequently we have proposed (Connes, 1978) a Fellgett–Griffin-type meter designed specifically for small velocity variations and the planetary search problem; it was to be a large Coudé-focus type device, many features of which will be reused here. The most important single point (and the only truly original one) was to be the use of a fused quartz fiber as *scrambler* (see Figure 1). Thus there was a good hope that a correlation meter (unavoidably based on a grating) could be made completely insensitive to seeing and guiding errors. Another feature of the proposal was to be the use of a FP etalon to provide the calibration check marks, which has distinct advantages over absorption lines. Since the instrument has never been built for lack of support, we do not know what the actual performance would be*. There is no reason to be today any more pessimistic about it; however, the proposal as it stood is clearly obsoleted by the new one to be made presently.

Here is a second, and totally different, line of approach. If we consider the way things are presently going within metrology, it is high time that 20th century celestial mechanics should jump into the frequency bandwagon, or get hooked up in some way. To some extent, this is already done: we do have a host of home-made oscillators gallivanting around in the solar system and wherever a solid surface at decent temperature offers support, we may anchor one of them, as already done in the case of Mars with striking results. For the stars, Sun included, there is no such hope: the best we can do is to make use of their own and sadly incoherent radiation. Fortunately, there is plenty of it. For instance, in the relativity experiment performed from the Martian Viking transmitter, the power received on Earth was of the order of 10^{-10} erg s^{-1} (Shapiro *et al.*, 1979); with the 'solar accelerometer' to be described here, it will be about 5 ergs s^{-1}. The very large

* However, an essentially similar velocity-meter, indeed built for planetary search, has just been described by Flint (1984); the system is practically complete but not yet tested.

ratio between the two figures offsets to some degree the considerably greater intrinsic accuracy of coherent techniques relative to incoherent ones*.

The first idea that comes to mind is to make use of heterodyne spectroscopy (Abbas *et al.*, 1975). This conceptually very simple technique provides the most direct possible link between any optical spectrum – astronomical or otherwise – and a laboratory checked frequency. Hence, it is unbeatable as far as the elimination of all systematic error goes. No full discussion is intended here; we shall merely recall that the technique suffers from very poor efficiency on two different counts. First, the heterodyne spectrometer may use fully the telescope beam only if the seeing disk is smaller than the Airy disk. In the visible, this means that everything goes as if the telescope had to be stopped down to a diameter of a few centimeters; this limitation disappears in space with a truly diffraction-limited telescope. However, the second one remains: the actual spectral bandwidth used is of the order of the detector bandwidth, i.e., a few GHz, which is an extremely small fraction of the visible spectral range – about 3×10^5 GHz. And even ideal, widely tunable, lasers applied to the problem do not improve the situation: they might permit to analyse a broad spectral range, but only one element at a time since they are scanning devices, and basic efficiency is not changed. Furthermore, all presently existing heterodyne spectrometers operate in the IR where the velocity information present is particularly poor (as shown in Section 4.2). So far, no attempt at determining a stellar radial velocity seems to have been made, and the fundamental (photon noise induced) velocity error has not even been computed. Still, a discussion of heterodyne spectroscopy potential in the planetary search context has been given by Deming (1983). The point mostly stressed is that it should prove a good tool for accurate line profile determinations, which are relevant to the problem. Altogether, this beautifully simple and radical solution is not a practical one.

Let us conclude this introduction. On one side we have, with heterodyne spectroscopy, a proven technique which is practically free of all systematic errors, but of exceedingly small efficiency. At the other extreme, we have with the radial velocity photometer a very efficient device, but so far with rather high-instrumental errors. Do we have here an incompatibility due to an act of God, or shall we suspect a human artifact? The new technique to be described now solves both problems at the same time. On one hand,

* *Coherent versus incoherent Doppler shift detection*: one should not forget that (at least in the laboratory), a technique which is fully incoherent is nearly matching optical interferometry: in the Katila and Riski (1981) experiment (an updated version of the classical 'weight of photons' measurement by Pound and Snider, 1965), the minimum relative detectable velocity between a pair of Mössbauer absorbers went down to about 10^{-7} cm s^{-1}. In the stellar accelerometer to be described here (also using incoherent detection) the stellar spectrum and corresponding mask function do play a role formally analogous to that of the Mössbauer pair; but the line finesse is enormously less, while the number of photons s^{-1} is much larger. The analogy may seem artificial, but it is much less so if one remembers another (far more modest) laboratory experiment which does fit nicely in between the two: small Doppler shifts between a pair of FP etalons were detected also in a fully incoherent manner (Connes *et al.*, 1962); from that experiment the present proposal derives in part. Still, the immense superiority of coherent techniques under astronomical conditions must be illustrated by a topical example: the proposed gravitational wave explanation of the 160 min solar oscillation. As shown by Anderson *et al.* (1984), Doppler tracking of a spacecraft is indeed a vastly more sensitive (and adequate) way of catching any passing GW than observing any solar surface feature.

we are able to demonstrate that it improves over the efficiency of the optical mask correlation spectrometer by getting closer to a fundamental photon count limit, so far never computed. On the other hand, it may emulate heterodyne spectroscopy for elimination of systematic errors, although this point will be fully settled by tests only. Both features together mean a potentially valuable tool for celestial mechanics. All of this development is made possible because of our starting remark: we do not have to measure *velocities* (as heterodyne spectroscopy or alkaline vapour cells do beautifully), only *accelerations*. Two brother instruments, respectively named solar and stellar accelerometer will be described. Both are totally specific: they are not intended to measure velocity, only the velocity changes of a given object, i.e., acceleration. Both are absolute; no empirical calibration is needed, and the measured acceleration is referred to the length and frequency primary standards. In both cases, the present paper will give only an abridged discussion; a fuller one is found in a longer unpublished paper with the same title, referred to as Connes (1983).

2. Absolute Solar Accelerometer

The heart of the system is a Fabry–Perot etalon with two different bandpasses separated by about one solar line width $\Delta\sigma$. Finesse and thickness of the FP are such that both half-widths are also equal to $\Delta\sigma$ (these conditions merely correspond to an optimum in the overall SNR; no systematic error results if they are not fulfilled). This may be achieved by using two beams at slightly different incidences, but the preferred method is that indicated on Figure 1. In this manner we actually have two closely interlinked etalons; from a single incident beam we get two separate outputs A and B which go to two different detectors.

This double FP (Figure 2) receives two geometrically coincident beams, and operates as a null device within two independent servo loops. The first beam is solar and comes from a small Sun-seeker through a fiber (which acts both as integrator and scrambler, see Section 5), and broad band filters. A grating spectrometer eliminates all orders but one, and two signals with intensities I_{AS} and I_{BS} corresponding to the two flanks of the solar line are produced by two detectors. From the difference $D_S = I_{AS} - I_{BS}$ the optical thickness of the FP is servo-controlled, either by mechanical displacement of one plate, or by pressure variation. The servo nulls D_s; then the two bandpasses follows the wavelength fluctuations of the solar line. One does not measure I_{AS} nor I_{BS}; the dynamic range of detectors or amplifiers (and the mechanical stability of the etalon) are irrelevant; zero drift is not, but may be taken care of by optical modulation.

A second beam from a tunable He–Ne laser at 6328 Å simultaneously goes through the etalon, making use of a different order of interference. The two beams are separated by the grating, and the laser beam proceeds to a second pair of detectors. A new difference $D_L = I_{AL} - I_{BL}$ is generated and controls the laser frequency by moving one of the cavity mirrors, mounted in a rather standard way on a piezoelectric ceramic. We again have $D_L \rightarrow 0$, and neither I_{AL}, I_{BL} nor the pressure in the etalon have to be measured. The laser frequency simply tracks the etalon bandpass, hence, the solar line.

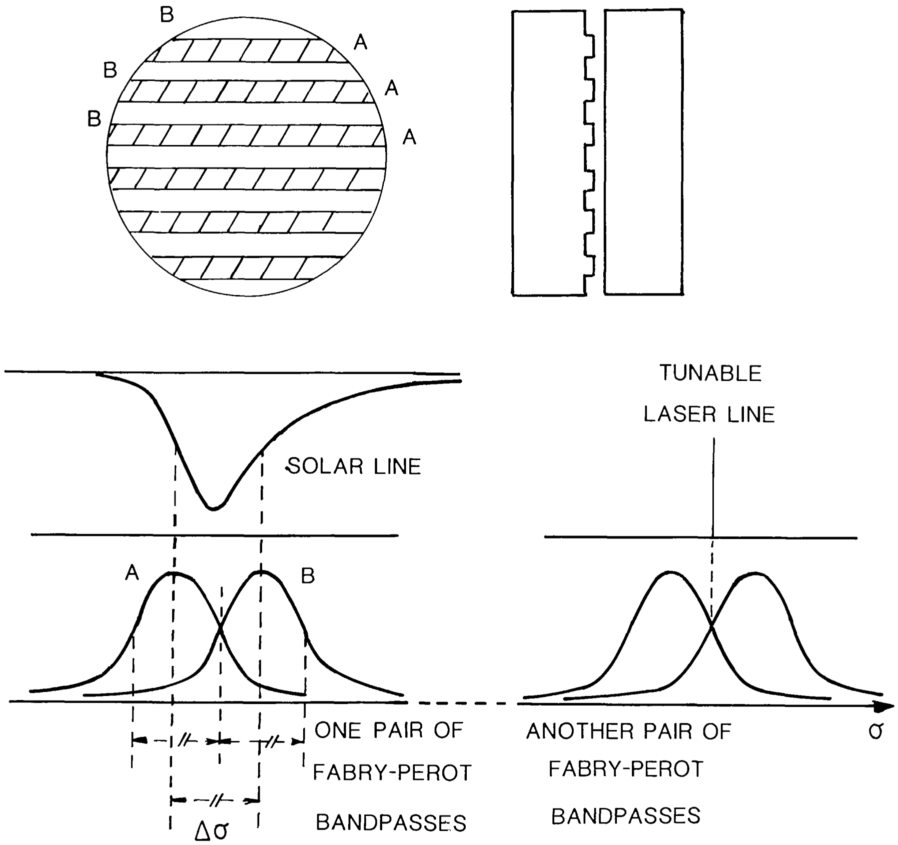

Fig. 1. *Below*: Positions and widths of FP bandpasses relative to solar and laser line, respectively. Neither the solar profile nor the FP one have to be known or symmetrical in any way. *Above*: One possible type of double FP. A thin layer ($\simeq$ 100 Å) of transparent material (e.g., SiO_2) is evaporated through a grid over one of the plates, before the reflective coatings; only the order of magnitude of the thickness must be right.

From now on, our basic problem (measuring wavelength fluctuations) has been transferred from the *incoherent* light domain to the *coherent* one: the tunable laser frequency is compared to that of a iodine stabilized one by mixing and beating, and the beat frequency is measured: this is the sole and final output of the whole experiment. Let λ_{S1} be the wavelength of some feature in the solar line profile (e.g., the CG) measured from the ground at epoch E, with a relative velocity V_1. The corresponding tunable laser wavelength is λ_{L1} and the FP thickness is is T_1; at epoch E_2 we have V_2, T_2, λ_{S2}, λ_{L2}. Thanks to the double servo action, the two orders of interference K_L, K_S have remained constant. Hence, $K_S = 2T_1/\lambda_{S1} = 2T_2/\lambda_{S2}$ and $K_L = 2T_1/\lambda_{L1} = 2T_2/\lambda_{L2}$; then $\lambda_{L1}/\lambda_{L2} = \lambda_{S1}/\lambda_{S2}$.

Let N_0 be the stabilized laser frequency, N_1, N_2, the two tunable laser frequencies, and $B_{N1} = N_1 - N_0$, $B_{N2} = N_2 - N_0$, the beat frequencies. We have

$$\frac{N_1}{N_2} = \frac{\lambda_{L1}}{\lambda_{L2}} = 1 - \frac{V_2 - V_1}{c} \;,$$

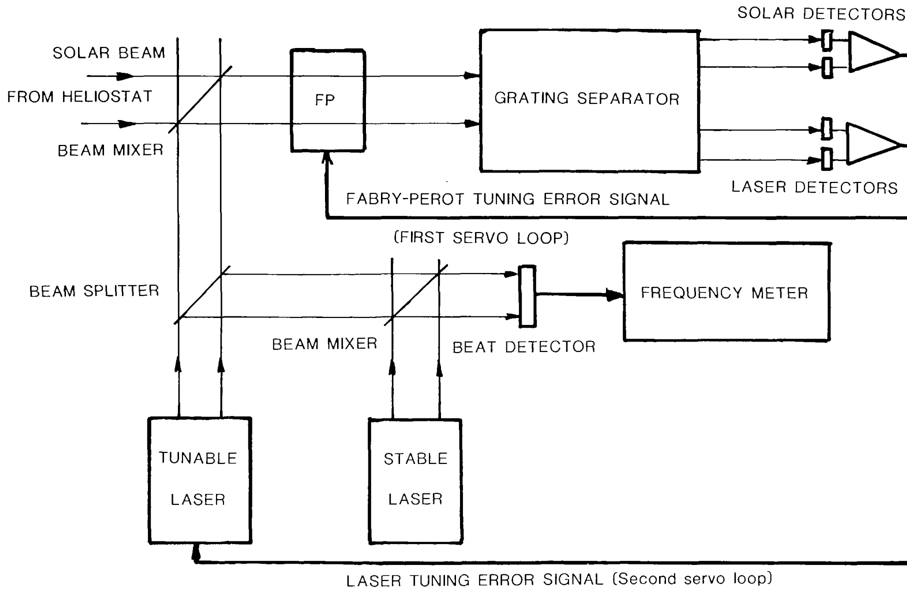

Fig. 2. Solar accelerometer block diagram. The grating separator actually play two different roles: first it separates the solar and laser beams and sends them to two pairs of different detectors; second it eliminates unwanted FP orders from the solar spectrum. One would use a double pass subtracting dispersion spectrometer, which provides both a trapezoidal bandpass (with almost ideally flat top), and high stray light rejection. The heliostat actually reduces to a small lens feeding the integrating-scrambling fiber; an image rotator may be added in front (see Connes, 1984).

and the wanted velocity change $V_2 - V_1$ is

$$V_2 - V_1 = c\,\frac{B_{N2} - B_{N1}}{N_0 + B_{N2}}\,. \tag{1}$$

The thickness of the FP has disappeared together with the solar wavelength: *we have to measure only the beat frequencies*. And we must know the velocity of light (which provides our new standard of length) and the frequency of the stabilized laser, which is measurable relative to even more stable standards, i.e., masers. The result expressed by Equation (1) may be directly compared with that obtained in Doppler radar, with, however, one essential difference: in Doppler tracking, absolute velocities, and not merely velocity changes are accessible. But, as far as accelerations are concerned, our method truly deserves that somewhat hackneyed epithet absolute.

Alternatively we may write

$$V_2 - V_1 = \lambda_0(B_{N2} - B_{N1})\left(1 + \frac{B_{N2}}{N_0}\right),$$

in which case the laser frequency appears only within a small corrective term, and we must know the laser wavelength λ_0; in the case of the convenient 6328 Å laser, this is presently a recommended secondary standard.

What does absolute mean in the present context, and what are we actually measuring? First, the fundamental viewpoint: we have dropped any possibility of measuring the velocities themselves (or the gravitational shift), and are concentrating on the acceleration between the observer and the whole visible solar hemisphere, as defined through any arbitrarily chosen solar line. Since all the Earth motions can be computed with extreme accuracy relative to the solar system center of the gravity, the final result is the same that would be recorded by a true absolute seismometer lying upon the so defined fictitious solar surface. Two connected remarks: we cannot hope for the same high degree of accuracy (which the instrument will make possible), if we attempt to isolate any small portion of the solar disk, because this introduces guiding errors. Second, the interpretation in terms of meaningful phenomena (e.g., oscillations) of this fictitious surface motion is clearly a complex matter, bringing in much of solar physics, not to be discussed here; we merely note that the highly informative sodium and potassium cells results suffer from the same limitations, and that the line cannot be chosen at will.

From a more practical side, we see that the instrument requires no calibration of any kind; solar line and instrumental profiles are utterly irrelevant. Consequently, when we want to extract the hoped-for solar oscillations, we do not have to compute a smoothed data curve and take the difference. The corresponding advantage is an important one. Ground-based solar data (except from the Pole) do present gaps due to intervening nights, and the corresponding power spectra are badly marred by numerous ghosts. We have shown, however, that all of these may be removed by a suitable numerical treatment (Connes and Connes, 1984); hence, this particular difficulty is not fundamental. A more serious one remains, common to all so far used devices which (as stressed in Section 1) do require calibration: each daily recording must be treated separately. The deleterious consequences are not too severe for the 5 min oscillations, but become progressively worse for longer periods, in particular for the potentially important g-mode range. The difficulty vanishes with an absolute accelerometer. The advantage may be understood by comparison with ACRIM (Wilson, 1979), an active solar radiometer flown on the SMM spacecraft which has produced remarkable seismic solar spectra (Woodard and Hudson, 1983), from global intensity fluctuations. Precisely because ACRIM is also an absolute device no calibration is required either; while the data do present orbital gaps, they can, nevertheless, be treated as one continuous record. The whole data treatment is simple and easily intelligible; by contrast, predicting all possible artifacts from the multiple calibration procedure of a non-absolute device is no simple task; the point will be pursued elsewhere.

As to the low-frequency drifts, they should in principle be limited only by the slow fluctuations of the iodine stabilized laser, which must be distinctly less than the reproductibility error between two separate lasers, presently about 10^{-11} (BIPM, 1979). Even more striking figures (10^{-14} for the Allan variance) are already being quoted for the ionized argon laser, also stabilized on iodine (Bordé *et al.*, 1980), but long term stability has not yet been checked. The velocity error corresponding to a 10^{-11} frequency drift would be 3 mm s^{-1}. Consequently, the system enables us to follow the velocity fluctuations of the Sun (in integrated light) for very long periods of time. Actually, indefinitely long: when the old laser goes the way of all flesh, we merely put in a young one, and have at most 3 mm s^{-1} error; or none at all if we have taken the obvious course of comparing the two in due time. Anyway, the ultimate limits will be set by the stability not of our laser, but of the primary frequency standard.

Still, absolute does not mean error free. Let us stick to some of the simplest points. First, how large is the beat frequency? The maximum velocity change to be measured during any given day is about $\pm$ 300 m s^{-1} relative to the meridian passage. If we are using the convenient 6328 Å He–Ne laser, with $N = 4.74 \times 10^{14}$ Hz, the required tuning range will be 950 MHz, which is just about what the Doppler width allows. Supposing the tunable laser frequency to be adjusted equal to N_0 at noon, the maximum beat frequency will be 475 MHz, easy for common photomultipliers and solid state detectors. However, the additional yearly variation is $\pm$ 500 m s^{-1}, which leads to a total range of $\pm$ 800 m s^{-1} or $\pm$ 1250 MHz; this is too much for the tunable laser. Hence, during the year, we shall have to use two or three slightly different FP spacings, which matters little: the result is still absolute. Other possible techniques are studied in Section 5.

The only *essential* limitation to accuracy comes from photon noise in the servo tracking of the solar line center. A standard calculation leads to a RMS error:

$$\delta V_{\text{RMS}} = \frac{\Delta V}{K} P_0^{-1/2},$$

where ΔV is the linewidth expressed in velocity; K, a numerical factor of the order of unity, function of line depth, and shape; and P_0, the mean number of counts in each of the two channels. Considering for instance the potassium line at 7699 Å, a 30 mm diameter etalon fed by a 1.5 mm fused silica fiber and a 20 mm diameter heliostat, a Reticon detector (which is conveniently matched to the grid-like double FP), and a 0.1 overall transmission, one finds $\delta V_{\text{RMS}} \simeq 3$ mm s^{-1} for a 1 s integration time. This figure gives no better than an order of magnitude; still it should be compared with the result of a corresponding calculation for the sodium vapour cell (Grec *et al.*, 1976) which gives $\delta V_{\text{RMS}} = 26$ cm s^{-1} for 1 s. The difference is due mainly to the relatively poor efficiency of the resonance process in making use of incident photons. This high SNR combined with the relatively low sampling rate required (of the order of 100 s/sampling interval for the so-called 5 min oscillations), means that our accelerometer could easily operate on many solar lines in a quasi-simultaneous mode, simply by switching the grating separator from line-to-line. The system is applicable from near UV to near IR. Systematic differences in seismic spectra might be found, e.g., because of changing limb darkening, and for other reasons.

Altogether we should not be so naive as to hope that secondary error causes will easily be reduced to the extremely low level of the fundamental ones. With heterodyne spectroscopy, frequencies only are *truly* involved. Here the final result is also in terms of frequencies; still, the whole device is an interferometric one, and of course we have been handling wavelengths all along, up to the point where the tunable laser is adjusted: all the usual failings of optical systems must be expected. For instance, extreme care will be needed to insure identical illumination of the FP by the solar and laser beams: considering standard plate surface errors, variable transverse shifts of one of the beams relative to the other would induce large-frequency errors. Consequently, the laser beam should also be sent through the scrambling fiber (see Section 5.1 and Figure 11). A more detailed study of the system is not intended in the present paper, where the solar accelerometer has been tackled mostly as a pedagogical device in order to introduce the conceptually far more complex stellar accelerometer.

3. Absolute Stellar Accelerometer

3.1. Principle of operation

The essential difference between the solar and stellar cases is that no single-line device gives any hope of success on stars, because there are simply not enough photons available, at least when we have in mind the same kind of high-resolution and low-noise seismic spectra we are getting used to from the Sun. Nevertheless, let us start from the solar accelerometer, which made use of a single line. We note that the chosen solar wavelength disappeared from the final result. This means that the small retuning of the FP needed to track the Doppler shift is *independent of the selected wavelength*. Hence, if we had the luck of finding a star whose spectrum exactly matched that of the FP, i.e., one with strictly periodic structure (constant wavenumber interval between lines), the accelerometer would operate correctly, however wide the spectral range. All lines would coincide with FP peaks; each peak would produce a signal; all such signals would add coherently, and the overall SNR would be exactly the one we may compute by adding energy from all lines.

Unfortunately, no such strict periodicity is practised by actual stars; a few lines would indeed coincide, most would not; the individual signals would add incoherently and the result would be useless. Moreover, the efficiency would again be very low, as in the FP spectrometer discussed in Section 1. Hence, the simple scheme that works well for the Sun is not good for stars. Due to this regrettable lack of collaboration from the part of Nature, far more sophistication (not counting hard cash) will be required from the experimenter; but he can still win the game. Why? Here is the essential reason in a nutshell: over any broad spectral range, the two spectra (stellar and interferometric) do not match. *But the shifts do.* We see intuitively that the problem must be soluble: if we were to observe simultaneously (with a suitable grating spectrometer plus detector array) the full stellar spectrum on the one hand, and the pure FP spectrum on the other from a white light source, *thus not passing stellar light through the FP*, we might in principle check position of all lines, stellar and FP, without hindrance from one to another. When radial velocity varies all stellar lines shift; we might then tune the FP until each and all of its own lines have matched that shift, a match that would be checked by a new set of measurements. Then, indeed the tunable laser frequency would again track the star velocity, and all stellar lines would have been made use of; furthermore, stellar light would not have to pass through a high finesse, poor efficiency etalon. The central

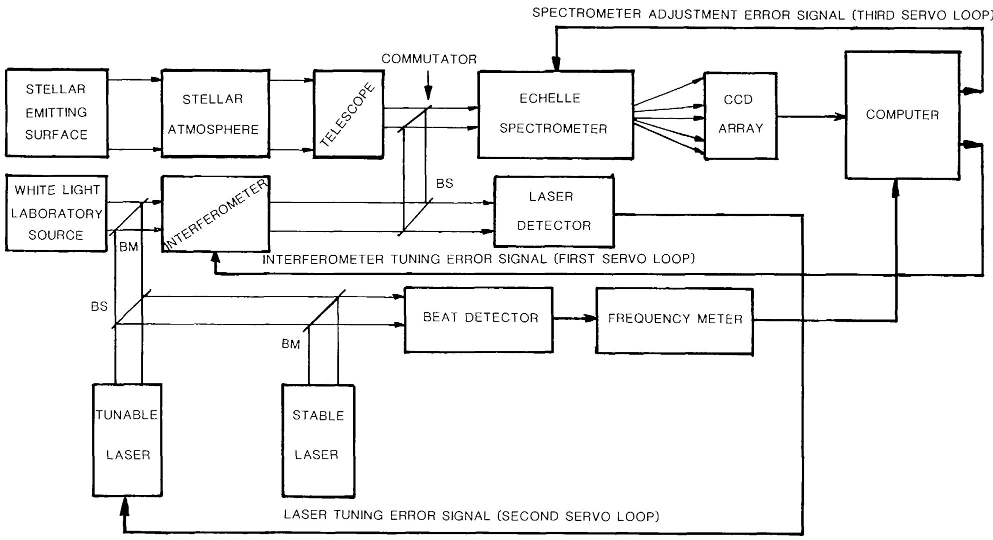

Fig. 3. Stellar accelerometer block diagram. BS and BM are beam splitters and beam mixers; the commutator is a rotating or flipping mirror. Note the complete symmetry between the stellar atmosphere and the interferometer: both must be thought of as similarly impressing calibration marks on a continuous spectrum, but the second, unlike the first, is under control. This symmetry is strictly preserved by the later treatment (except that a larger fraction of the commutator cycle is spent on the star), and illustrated by the definition of the fictitious Fabry–Perot velocity. There is no need for the laboratory white light source to emulate the star temperature; any difference can be compensated in the data handling by using the weighting function $F(\sigma)$. Unlike in the solar accelerometer case, the FP is a plain one (no evaporated strips required) with a single single output; generation of the laser tuning signal is then slightly different (Connes, 1983). There is no stray light from the laser and white light source to the stellar spectrum; they are never observed simultaneously.

difficulty is to implement this scheme with a device of acceptable optical complexity, not reintroducing spectrometer errors, nor requiring unacceptable amounts of computer time. Let us show that a workable one may be built-up with an echelle spectrograph used in a time-sharing mode, i.e., measuring alternatively the FP and stellar spectra.

The heart of the stellar accelerometer is given by Figure 3. Star light and the beam from a white light source which has passed through a FP etalon, are alternately sent into an echelle spectrometer by a reflecting rotating chopper. The spectrometer is similar, at least in general outline, to CORAVEL, or to the one proposed by Connes (1978). However, there is no mask and the output falls on a two-dimensional detector. Any type which may be hooked up to a computer is adequate; presently, the most suitable one seems to be the CCD array. Signals are stored, then treated by the computer whose output is converted to analog and constitutes a self-nulling error signal; this adjusts FP spacing, until both spectra are similarly shifted from one observation to the next. Simultaneously, a tunable laser again tracks the FP and a beat signal is generated from a stabilized laser.

Basically, the role of the spectrometer – CCD array-computer system is that of a null-checking device: it must merely control the identity of both shifts, not measure them. No calibration is involved anywhere. One does not either measure wavenumbers, nor wavenumber changes, nor line positions in any way; detector array shape and stability are in principle irrelevant, and the array may be replaced by a new one, even a different one, at will. From these features we may at least fondly hope for a high degree of cancellation for the usual grating spectrometer errors; this will be shown in Section 3.3.

While this double servo control scheme appears generally similar to the already described one, there are nevertheless important differences, particularly in the relative roles of spectrometer and FP interferometer. In the solar case, the FP *by itself* must provide the needed resolving power, hence has linewidth approximately matched to the solar one. The function of the spectrometer is merely to eliminate unwanted orders. Consequently, the FP finesse has to be as *high* as feasible, which makes spectrometer resolution lower and eases its design; starting from the solar potassium line at 7699 Å, with a width of about 6 km s^{-1} and supposing a finesse of 30, the FP spacing is about 0.6 mm and the spectrometer resolution 1600. In the stellar case, however, the spectrometer-CCD system must correctly sample the star spectrum, i.e., provide the necessary resolution. Then, FP linewidth and separation must be chosen in order to provide in the spectrum as many checkmarks as possible; but these must be still resolvable by the spectrometer. Hence, finesse must be as *low* as possible and FP resolution is chosen starting from the spectrometer one. Taking a spectrometer resolution of 10^5 (which is desirable, see Figure 9) and a finesse in the range of 2 to 3 (which means low contrast, but that is no drawback), one is led to FP spacings in the range of 5 to 20 mm (see Connes, 1983).

On closer examination, once does find a second difference with the operation of the solar accelerometer. In that case, in principle just two photocells were sufficient to observe both flanks of the solar line. This pair *by itself* constituted the null checking system. No interference fringe motion was involved anywhere and FP retuning did restore, for each new solar velocity, exact equality of illumination for both cells. Here the situation is more complex. So far we have supposed the spectrometer and CCD to be fixed, as in a spectrograph. If this is true, then the spectral lines move across the CCD; in the worst case the maximum displacement corresponds to a 60 km s^{-1} velocity change, i.e., many times the linewidth. Hence, the CCD *by itself* does not check a null. Of course the displacement is precisely the same for both spectra; still, these are not identical and because of various non-fundamental but neverthless troublesome detector phenomena (Geary, 1981), one may fear that such a large line motion might reintroduce systematic effects. All of them might be calibrated out. A better procedure (and one more in keeping with our general philosophy) is to null the spectrum-CCD relative displacement itself. There is no exact solution: the problem is the same as that of the Fellgett optical correlation meter with a rigid mask. However, fairly good approximate ones are available: in a typical case the residual displacement from the 60 km s^{-1} maximum possible change is only 30 m s^{-1}, i.e., a very small fraction of the linewidth; as we shall see (Section 3.3.4) this is fully adequate for our purpose. Hence, we introduce at this point a third (or auxiliary) servo-loop, which does not interact with or perturb in any way the other two: the spectrometer is readjusted until the shift of spectral lines on the CCD is approximately nulled. Since this result holds good for both spectra simultaneously, the condition may be checked by using only the FP spectrum: operation of this third loop does *not* require stellar observations. It remains auxiliary in the sense that it could be dispensed with, while the measurement would remain absolute; and its own errors (small spectrometer misadjustments) fully disappear from the final results.

One easily shows that the whole procedure indeed leads to absolute accelerations; the full demonstration is given by Connes (1983). The stellar and Fabry–Perot spectra are called $A_S(\sigma)$ and $A_F(\sigma)$, respectively, and handled in exactly the same manner*. When passing from epochs 1 to 2, one is led to define for the two spectra two 'stretching factors' $\Omega_S = 1 - (V_{S2} - V_{S1})/c$, where V_S is the stellar velocity and $\Omega_F = T_2/T_1$ where T is the FP thickness. To make symmetry complete, it is convenient to introduce a fictitious 'Fabry–Perot velocity' V_F such that $\Omega_F = 1 - (V_{F2} - V_{F1})/c$. And the essence of the whole method is to adjust T until $\Omega_F \equiv \Omega$ and $V_F \equiv V_S$ where the sign $\equiv$ means that the result is achieved simultaneously at all wavenumbers. This is of course is easier said than done; Section 3.3 will be devoted to describing the generation of the FP error signal which produces this result. However, when this is achieved, we find the same result as in the solar case, i.e., Equation (1). Consequently, we get the considerable advantage (relative to all standard methods of measuring stellar velocity changes), that the enormously larger Earth induced terms are cleanly removable at a single stroke by operations involving frequencies only; these are for all practical purposes here, infinitely accurate.

A last remark: at first glance one might fear non-essential but still bothersome differences in the measurements of the two velocities because while the FP spectrum is periodic, the stellar one is more or less random. However, the FP spectrum is periodic indeed, but only in wavenumbers, definitely not in pixels, because of nonlinear dispersion. Hence, as seen by the CCD, and throughout data handling, the two spectra do not look essentially different.

3.2. Observing procedure

3.2.1. *Preliminary Observation at Epoch 0*

The star is observed for the first time. The stellar and FP spectra are recorded in order to establish the shape of both; there is yet no question of measuring acceleration. During the observation, the FP is kept stable by its own servo loop, i.e., the laser is locked to some constant beat frequency. The mean radial velocity during epoch 0 is chosen as origin of the velocity scale for all later observations, i.e., one arbitrarily sets $V_0 = 0$. The resulting spectra are called $A_F(\sigma)$ and $A_S(\sigma)$ and stored for later use.

* We prefer to use as a variable the wavenumber σ, because (as in all problems involving interference) the results are simpler: both the echelle and the FP free spectral ranges are constant. Of course, essentially the same results would be obtained using λ instead.

3.2.2. *In Between Epochs*

The star and FP will now be observed at successive epochs $E_1, E_2, \ldots, E_n$ in order to establish the velocity changes $V_1, V_2, \ldots, V_n$. These consist of a small unknown stellar term (the goal of the experiment) plus a very much larger Earth induced one. Before any observation begins, this second term is accurately computed from Earth motion constants and stellar coordinates. The corresponding frequency change is predicted, and the FP and laser are tuned until that new frequency is generated. Hence, at the start of each observation, the FP spacing is already *almost* that which will null $V_F - V_S$. Simultaneously, the third loop has been actuated (from the observed FP spectrum), and the spectrometer readjusted until the lineshift on the CCD is minimized. This means that the spectrometer is actually locked to the FP, i.e., to the laser; hence, any spectrometer drift in between epochs has been taken care of before observation starts.

3.2.3. *Epoch n Observation*

During the observation, the FP is kept stable by this servo lock and the laser frequency remains fixed at the preset figure. No feedback from the stellar spectrum to the FP is yet used. However, both spectra are successively observed thanks to the commutator. The integrated spectra available at the end of epoch *n* are designated $A_{Fn}(\sigma)$ and $A_{Sn}(\sigma)$. Since the noise contribution of the FP channel is expected to be negligible, the greater part by far of the commutator cycle is spent in observing the star. The choice of the integration time T results from a compromise between several factors; it would range from a few time 10 s for bright stars, to a few times 10^3 s on faint ones (Connes, 1983).

3.2.4. *Processing at the End of Epoch n*

The two spectra are immediately processed in order to generate the two 'velocity difference' signals. A small non-zero $V_{Fn} - V_{Sn}$ is found, due to the unknown stellar term. From this just measured difference plus the pre-computed Earth velocity change from E_n to E_{n+1}, one predicts the laser frequency that will presumably null that difference during the next epoch. The FP is tuned until that frequency is produced; simultaneously the third loop is also actuated, and the spectrometer readjusted in order to cancel lineshift during the next epoch.

3.2.5. *Succession of Epochs*

Then the next integration cycle is performed and the entire operation repeated. From now on, during the whole time allocated to observations, beat frequency tracks Doppler shift, and feedback continually attempts to null velocity difference between the two spectra with a lag time equal to one integration period, which is normal for any servo loop under digital control. The net output of the whole experiment is made up of two sets of complementary records. The first is that of the successive beat frequencies, which have remained constant during each epoch; these represent the stellar velocities as predicted for each epoch from the results of the preceding one. The second consists of the measured differences. From both, the actual radial velocity history of the star may be built up *a posteriori*, of course as seen through integration windows of duration T.

3.3. Generation of velocity difference signals

Let us show how, starting from the two spectra as measured by the CCD array, two distinct velocity difference signals may be generated; these will be used to tune the FP interferometer and readjust the spectrometer, respectively. There is no unique solution to the problem. The one to be described here, involves the use of 'mask functions' so called because they play a role analogous to that of the physical mask in the Fellgett–Griffin correlation meter. In the present case two of these masks are needed, one for the FP and one for the star. They are generated from the epoch 0 results; at any later epoch, the spectral shifts relative to these masks are detected, and the corresponding velocity difference signals computed. The most remarkable feature of this treatment is that it will be shown later (Section 4.1) to be optimal as far as basic photon or detector noise limitations go.

The full discussion of the method, being somewhat lengthy, has been given by Connes (1983); only the main points will be covered here. In particular, we shall make four simplifying assumptions; dropping them can be shown to introduce no systematic errors and only a moderate loss of signal to noise ratio. These four assumptions are needed in order to present a demonstration based on the ideal stellar and FP spectra, expressed as functions of true wavenumbers, while the actual computation will be made from spectral samples as provided the spectrometer.

(a) Instrumental resolution is supposed infinite: i.e., the spectrometer is able to provide the *true* spectra $A_F(\sigma)$ and $A_S(\sigma)$.

(b) Discrete sampling by the detector array is ignored: i.e., the *continuous* functions $A_F(\sigma)$ and $A_S(\sigma)$ are supposed available.

(c) Spectrometer dispersion is supposed constant within each echelle order; this is approximately true for the narrow field actually used with a high-order echelle.

(d) One further supposes that the third loop may readjust the spectrometer in such a way that Doppler displacements are cancelled *simultaneous* for all lines in the spectral range, which is only approximate, as stressed above. Assumptions (c) and (d) may appear coarse at first glance; the saving grace comes from the rigourously identical approximation involved for the two spectra which, as we shall see, are handled in exactly the same manner.

We consider Figure 4 which presents an arbitrary spectrum $A(\sigma)$; this may mean either $A_F(\sigma)$ or $A_S(\sigma)$. Intentionally, no line profile is illustrated anywhere within the spectral range $\sum$; indeed, in the technique to be proposed here, there is at no time any operation involving the ritual setting-on-a-line traditionally involved in Doppler studies, ever since Huggins (1868). This remark does not contradict the obvious fact that most of the velocity information present in the spectra is given by lines, particularly the sharp and deep ones; this will be confirmed by our fully general analysis of the noise problem in Section 4.1. However, it is precisely because we *refuse to pose the problem in terms of lines* that we shall be able to formulate this analysis, and prove the treatment presently discussed to be optimal.

$A(\sigma)$ is what we observe at epoch 0, when the mask is built up; at epoch n the profile has stretched to $A_n(\sigma)$. Let $\delta V_n = V_n - V_0$ be the velocity change; this may be either the

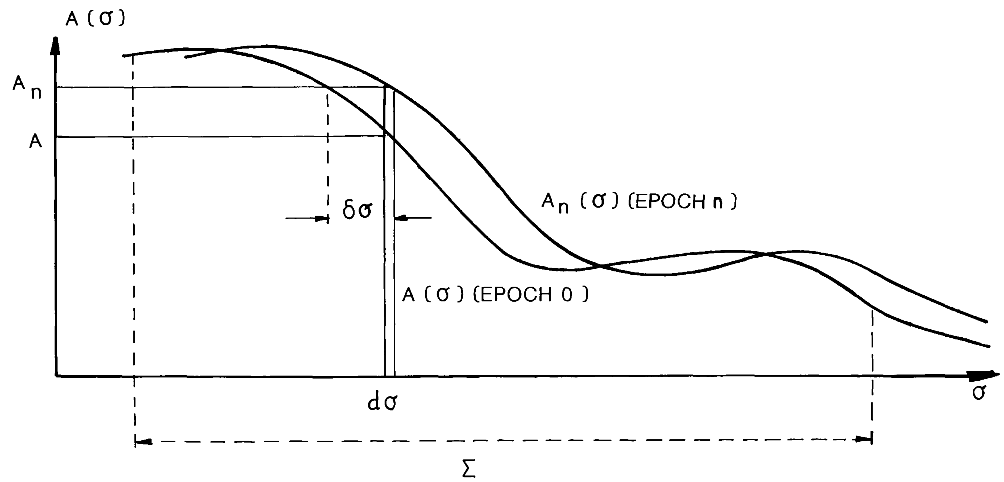

Fig. 4. An arbitrary spectral profile at epoch 0 and epoch n; the intensity change is measured within a given dσ slice.

true stellar velocity or the fictitious FP one. The corresponding wavenumber shift is $\delta\sigma_n = \sigma\delta V_n$. The observable intensity change* at some given wavenumber σ is given by

$$A_n(\sigma) - A(\sigma) \simeq \frac{\partial A}{\partial \sigma}\, \delta\sigma_n = \frac{\delta V_n}{c}\, \sigma\, \frac{\partial A}{\partial \sigma}\, . \tag{2}$$

Inversely, we may (at least in principle) obtain the desired velocity change δV_n from just one measurement of intensity change within dσ. Of course, in the presence of various errors (in particular those due to noise), we must make use of the whole available spectral range Σ and average results for all slices dσ. This may be done in various ways. Let us define a 'mask function'

$$M(\sigma) = \frac{\partial A}{\partial \sigma}\, F(\sigma)\, , \tag{3}$$

where $F(\sigma)$ is a weighting function which may play different roles; for instance it can be used to eliminate unwanted spectral regions (e.g., contaminated by telluric lines). However, we will show in Section 4 that when pure photon noise is considered, there is an optimum choice of $F(\sigma)$ which is then a function of $A(\sigma)$ alone.

* This result is true in the first order only and seems to require that the shift must remain small compared with the linewidths. However, we have already noticed that the maximum velocity change is 60 km s^{-1}, while the linewidth is about 6 km s^{-1} (solar case). Nevertheless, the complete treatment given in Connes (1983) shows that the approximation used in Equation (2) remains adequate under all circumstances. This is because in practice the intensities A and A_n are not measured and compared for a *given wavenumber* in the laboratory frame, but for a *given pixel*, and (thanks to the third loop) these almost exactly follow the lines. Then the residual shifts are indeed always small compared to be the linewidth.

Then we consider the integrals

$$I_n = \frac{1}{\Sigma} \int_{\Sigma} [A_n(\sigma) - A(\sigma)] M(\sigma)\, d\sigma \tag{4}$$

and

$$I_0 = \frac{1}{\Sigma} \int_{\Sigma} \sigma \frac{\partial A}{\partial \sigma} M(\sigma)\, d\sigma . \tag{5}$$

Then

$$\frac{\delta V_n}{c} = \frac{In}{I_0} = \frac{\langle (A_n - A) M \rangle}{\langle \sigma (\partial A / \partial \sigma) M \rangle} , \tag{6}$$

where $\langle\ \rangle$ denotes spectral averages over the range Σ.

Equation (6) gives the velocity change δV_n as measured from the full spectral range Σ. Now the generation of velocity difference signals may be summarized: after epoch O, $A(\sigma)$ is known; one computes $\partial A/\partial \sigma$, selects a suitable mask $M(\sigma)$ and computes once for all integral I_0, which plays the role of a mere normalizing coefficient. After epoch n, one has to compute (in real time) the integral I_n. These are of course numerical integrations performed from the set of samples given by the CCD. Exactly the same operation is performed for the star and the FP; thus, two integrals I_n and two I_0 have to be computed. Finally, we get the two velocity differences δV_{Fn} and δB_{Sn}.

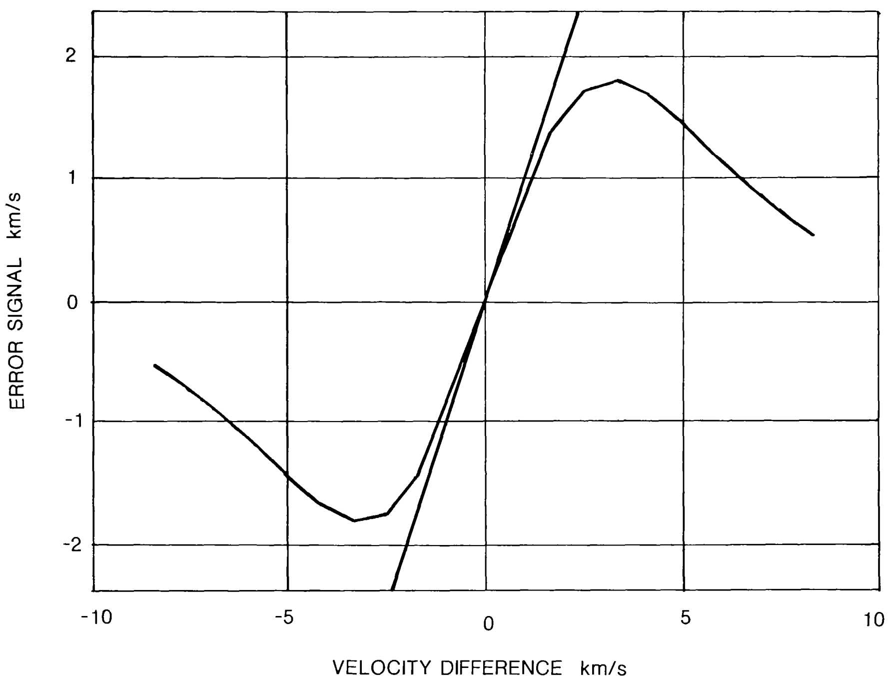

Fig. 5. The error signal δV_n numerically computed by Equation (6), starting from the *Solar Atlas* 3600 Å spectral range; abscissa is a numerically simulated velocity difference $V_n - V_0$ between epoch 0 and epoch n, with 800 m s^{-1} increments, corresponding to Atlas sampling interval. The tangent at origin corresponds to $\delta V_n \equiv V_n - V_0$. The third servo loop will always react in such a way that the velocity difference between the epoch n spectrum and the epoch 0 recorded mask, is well within the linear region.

How are they used to actuate the first and third servo loops? The full explanation (again somewhat lengthy) is to be found by Connes (1983). Suffice to say here that from the measured δV_{Fn}, the slight shifting of the FP spectrum at epoch n relative to its mask function is deduced; this would be zero if the spectrometer adjustment (checked at epoch $n - 1$) had remained perfect. Hence, an analog error signal is built up and applied to the spectrometer; no great accuracy is required because the crucial quantity is $\Delta_n = \delta V_{Sn} - \delta V_{Fn}$ which is independent of spectrometer adjustment. This last quantity represents the difference in the two velocities (and stretching factors) which arises from an imperfect FP spacing, resulting in turn from the imperfect stellar velocity prediction for epoch n, performed at epoch $n - 1$. Again an analog signal is built up and fed to the FP until Δ_n is nulled. There are several alternate procedures, in no way essentially different; one may, for instance, find more convenient to keep the FP continually locked at some preset beat frequency, and to compute from Δ_n the needed frequency change.

The consequences of our four initial simplifying assumptions are discussed at length by Connes (1983); only the conclusions need be given here:

(a) If we take into account finite spectrometer resolution, the only result is a reduction of the overall SNR, to be studied numerically in Section 4.2 and presented on Figure 9. The absolute character of the result is wholly independent of spectrometer performance; for instance the instrumental line shape does not have to be symmetrical. One is even able to prove that an *asymmetrically changing* ILS in between epochs (due for instance to slow warping of optical elements), introduces no first-order error; this is a very important result*.

(b) Any detector array will provide the spectrum in sampled form only which involves a second approximation. Again it is possible to show that no systematic error results. With CCD arrays, if one neglects the small lost space in between pixels, there is no further loss of SNR beyond that implied by the finite resolution. The computation involves one multiplication and one addition per sample; it is apparent that very small computers will be able to handle the computing load without appreciably increasing the servo lag time. Of course, the time required for merely reading the pixels must be added.

(c) Nonlinear dispersion within each echelle order is next taken into account. The final result is simply what we would have recorded without such distortion, but on a different star in which the lines would have been slightly sharper and closer on one side of each order, and less so on the other. There is neither error nor loss of SNR.

(d) Consequences of the fourth simplification are again unimportant although more

* The two underlying assumptions are: (1) There is no drift in between the two parts of the commutator cycle. (2) The spectrometer ILS is the same for both beams. The first problem may be cured by an additional dose of servo control; the second is solved by the scrambler, which ensures identity of illumination for both beams. Altogether, the demonstration given by Connes (1983) plus the use of a scrambler mean that the main advantage of the circular symmetry interferometers (Michelson or FP) for line position measurements (resulting from the second-order variation of path difference versus beam incidence) is so to say transferred to the grating spectrometer when our technique is used. It is relevant to quote at this point a recent result by Guelachvili (1981): a Fourier spectrometer has been operated in a time-sharing mode (much as proposed here), with two separate beams and a commutator. The system has been able to detect absorption lineshifts equivalent to a 15 cm s^{-1} radial velocity change.

difficult to explain without a full demonstration. In the Fellgett–Griffin correlation meter, which makes use of a rigid mask built up from a star spectrum photograph taken at some radial velocity, a slight mismatch arises between mask and spectrum at any different velocity. Fellgett (1955) has suggested this could be much reduced by moving two spectrometer elements simultaneously. This has been implemented with good results by Walraven and Walraven (1972), combining input slit and mask translations. Essentially the same technique was to be used by Connes (1978). Residual shifts will arise because (1) the use of two motions only gives an approximate solution; (2) the motions themselves will not be perfectly accurate. The resulting wavelength dependent mismatch should be of the order of a few times 10 m s^{-1}.

In these essentially mechanical devices the problem is a critical one because the velocity is indeed measured from mask motion. In the stellar accelerometer we must expect those same shifts between spectrum and mask function. However, the critical error signal (that tuning the FP) is not given by δV_{Fn} but by $\Delta_n = \delta V_{Sn} - \delta V_{Fn}$. Since the mismatch is exactly the same in both cases, it disappears fully from the final result. There is no direct effect at all. Connes (1983) discusses the possibility of second-order effects, due to combined mismatch and local stellar spectrum intensity changes; these appear negligible.

4. The Fundamental Noise Limitation

4.1. General analysis

A general treatment of basic photon count limitations in radial velocities measurements is given here. It applies directly to our stellar accelerometer, which will be seen indeed is the first so far proposed device with the ability to approach closely fundamental limits in this context. However, the demonstration itself is of more general importance. The starting point may be found in some pregnant (and widely unheeded) hints given by Fellgett (1956) in his fundamental paper on the subject; still, the accent was mostly on the *information-handling economy* brought in by the correlation meter concept. Today, we have detector arrays and much faster computers; and we must shoot for the *higher accuracies* promised by absolute accelerometry, because of hopefully reduced systematic errors.

The full demonstration is given by Connes (1983); here, only the essence of the argument is preserved. Let $A(\sigma)$ be the true stellar spectrum profile, as could, in principle, be determined from an infinite length observation at zero radial velocity. (In the present section, the subscript s may be dropped, since the FP is not considered.) Let us next consider a set of similar observations, all performed at the same zero velocity, and all lasting for the same time T. Each corresponds to a separate epoch n; in each case the measured intensity will be affected by a random error due to the finite and fluctuating photoelectron count. While there has been no effective velocity change in between epochs, a spurious one will nevertheless be found if we try to compare $A_n(\sigma)$, the noise perturbed profile measured at epoch n, with $A(\sigma)$. We are again considering an ideal spectrometer (infinite resolution, etc.).

Then, from Figure 4 and Equation (2) and considering solely the infinitesimal slice $d\sigma$, we have

$$\frac{\delta V_n(\sigma)}{c} = \frac{A_n(\sigma) - A(\sigma)}{\sigma(\partial A/\partial\sigma)},$$

where $\delta V_n(\sigma)$ is the wholly spurious noise-induced velocity that would be computed from epoch n results if only the infinitesimal slice $d\sigma$ were to be used. Next, we must treat all $d\sigma$ slices as providing fully-independent results, as far as noise is concerned. Then the rule for optimum weighting is that each weight must be proportional to the inverse square of the individual RMS error. There are two principal (but somewhat ideal) cases, those of pure photon noise, and pure detector noise. We begin by treating the first, which obviously corresponds to the best feasible accuracy. Let P_{AV} be the average monochromatic stellar brightness across range Σ, expressed in photons $s^1\ cm^2\ cm^{-1}$; from this we may compute the number of photons available within each $d\sigma$ and the corresponding $\delta V_n(\sigma)$ errors. Next, we apply optimal weights and we reach again Equation (6) provided, we take

$$M(\sigma) = \frac{\sigma}{A}\frac{\partial A}{\partial\sigma} \quad \text{and} \quad F(\sigma) = \frac{\sigma}{A}\,. \tag{8}$$

Hence, the use of a suitably weighted mask function *is an optimal technique, as far as basic SNR is concerned.* We are now able to compute $(\delta V_\Sigma)_{\mathrm{RMS}}$, i.e., the RMS spurious velocity determined from the whole spectral range Σ, relative to a large number of successive independent observations. We introduce

$$\bar{N}_\Sigma = P_{AV} S T \alpha \Sigma\,,$$

where S is the telescope collecting area; T, the integration time per observation; α, the overall efficiency; then $\bar{N}_\Sigma$ is the total mean number of photons collected during T from the whole range Σ. We also introduce

$$Q_P = \left[\frac{\left\langle \frac{1}{A}\left(\sigma\frac{\partial A}{\partial\sigma}\right)^2\right\rangle}{\langle A\rangle}\right]^{1/2} = \left[\frac{\langle AM^2\rangle}{\langle A\rangle}\right]^{1/2}, \tag{10}$$

which is a *quality factor*, defined for the spectral range Σ; it is independent of the actual size of Σ, but function of the spectral profile within Σ. It is valid in the pure photon noise case only, and will be explicitly studied a little later. Then we find

$$\frac{(\delta V)_{\mathrm{RMS}}}{c} = \bar{N}_\Sigma^{-1/2}\, Q_P^{-1}\,, \tag{11}$$

which solves the problem; $\bar{N}_\Sigma$ depends on stellar magnitude and instrumental light collection efficiency, and Q_P on stellar spectral type only.

Let us next consider the *detector noise case*. The fluctuations of A_n are now independent of A. Let us call $\bar{N}_D$ the new mean number of electrons counted, still for slice $d\sigma$; this may be either an actual or an equivalent number (i.e., producing the same RMS fluctuations). For pure detector noise we must have $\bar{N}_D \gg \bar{N}_\Sigma$ anywhere within $\sum$ and for the present purpose, we may characterize the detector by a factor $k_D = \bar{N}_D/N_{AV} \gg 1$ where $N_{AV} = P_{AV} ST\alpha \, d\sigma$ is simply the mean number of photoelectrons for a slice $d\sigma$ in which $A(\sigma) = A_{AV}$. Then, if we follow exactly the same procedure, the new mask function is

$$M(\sigma) = \sigma \frac{\partial A}{\partial \sigma}; \tag{12}$$

and instead of (11) we find that

$$\frac{(\delta V_\Sigma)_{\mathrm{RMS}}}{c} = \left[\frac{\bar{N}_\Sigma}{k_D}\right]^{-1/2} Q_D^{-1}, \tag{13}$$

where $\bar{N}_\Sigma$ is still given by (9), but with a new quality factor

$$Q_D = \frac{\left[\left\langle\left(\sigma \frac{\partial A}{\partial \sigma}\right)^2\right\rangle\right]^{1/2}}{\langle A \rangle} = \frac{[\langle M^2 \rangle]^{1/2}}{\langle A \rangle}, \tag{14}$$

valid in the *pure detector noise case.*

The mask functions and quality factors are easily computed when the spectrum has a simple analytical shape. For instance Figures 6 and 7 give the result in the unphysical

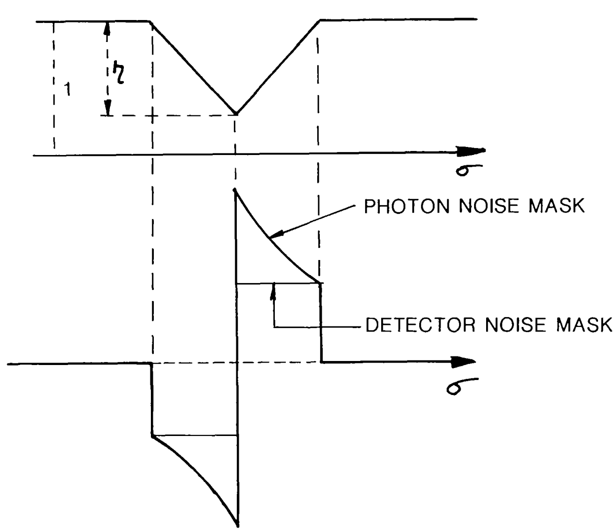

Fig. 6. A triangular absorption line with depth η and the corresponding optimal masks, computed from Equations (8) and (12) in the pure photon noise and pure detector noise cases. The photon noise mask profile is hyperbolic.

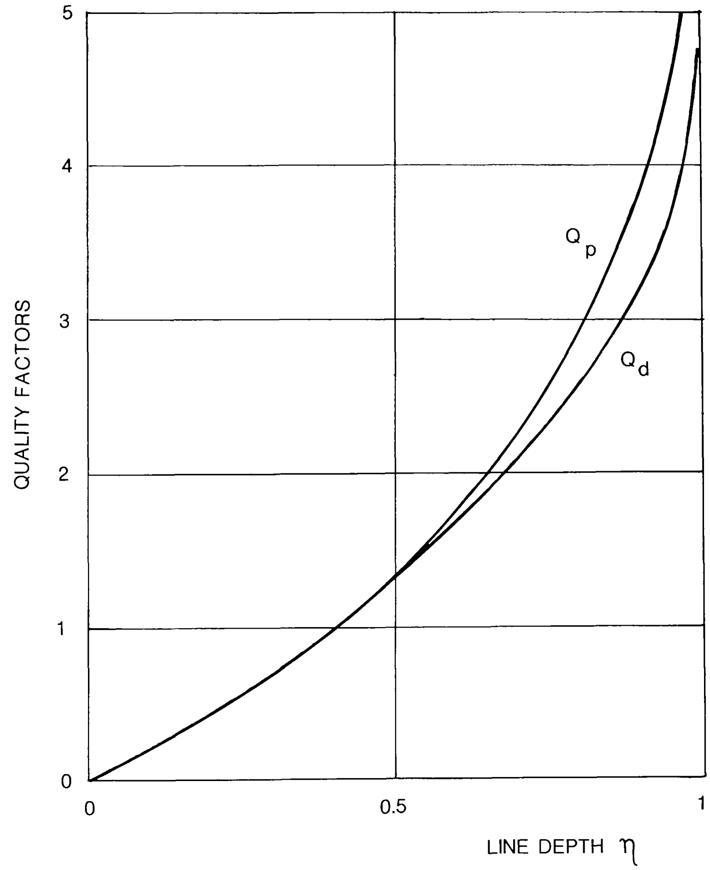

Fig. 7. The analytically-computed quality factors corresponding to the Figure 6 line shape. Both become infinite when $\eta \rightarrow 1$; this comes from the unphysical assumption of finite slope at zero intensity. Both are almost equivalent when the line depth is small; Q_P always lies above. Q_D.

but illustrative case of just one absorption line with a triangular shape. We next ask the question: What is the greatest possible quality factor for a spectral range densely packed with sharp lines? Strictly speaking, no absolute maximum can be defined since, as just shown, the result will be acutely dependent on the profile assumed, particularly near line tips if these go down to the zero level. The simplest and most sensible approach involves computing Q_P and Q_D for a pure harmonic channelled spectrum $A(\sigma) = \cos^2(\pi\sigma/\sigma_F)$ where σ_F designates the wavenumber period. We assume the spectral range to be small, i.e., σ differs little from a median value σ_m with $\Sigma/\sigma_m \ll 1$; however, $\Sigma/\sigma_F \gg 1$, i.e., the range still contains a large number of 'lines'. We introduce the 'line' finesse $F_i = \sigma_m/(\sigma_F/2)$, where $\sigma_F/2$ is the half-intensity width. We find that

$$(Q_P)_{CH} = \pi F_i \quad \text{and} \quad (Q_D)_{CH} = (Q_P)_{CH}/\sqrt{2}\,.$$

Both factors are independent of Σ; as expected Q_P is better than Q_D, and for this particular spectrum the use of the corresponding optimal mask would lead (with pure photon noise) to a reduction by a factor of 2 of the observing time. However, this is only true for the unit contrast spectrum postulated; from the results of Figure 7, we easily predict that for a low contrast channelled spectrum $(Q_P)_{CH}/(Q_D)_{CH} \rightarrow 1$ when contrast decreases.

Supposing $F_i = 5 \times 10^4$ as for the Sun, we find $(Q_P)_{CH} = 1.6 \times 10^5$, and $(Q_D)_{CH} = 1.1 \times 10^2$. These results might easily be refined by closely packing lines with true stellar profiles, but the figures would be of mere academic interest compared with the quality factors numerically computed from actual stellar spectra and discussed next.

4.2. Numerical computations for actual stars

4.2.1. *Quality Factors*

Our quality factors may be numerically computed for any star type for which we have a high-resolution spectrum which shows well the line profiles. We have used the *Solar Atlas* of Delbouille *et al.* (1973), the *Arcturus and Procyon Atlases* of Griffin and Griffin (1968, 1973), and the *Sirius Atlas* of Kurucs and Furenlid.

Since all these atlases are plotted with linear wavelength scales and sampled with a constant $\Delta\lambda$ interval, our computations have used λ as a variable, which matters little: identical expressions for Q_P and Q_D would have been obtained using λ instead of σ. However, P_{AV} will now represent a number of photons s^{-1} cm^{-2} $Å^{-1}$ and indeed this is what we find in astrophysical tables. Because the line density in these spectra changes considerably from violet to red, it is more meaningful to compute the Q factors separately for small ranges, and we have used 100 Å slices.

The most complete study has been made in the case of the solar spectrum, from 3600 to 8000 Å, i.e., the entire Atlas range. One has everywhere $Q_P/Q_D > 1$, but the ratio is distinctly smaller than $\sqrt{2}$, which is what we might expect considering that lines do not go down to the base level. The largest figures are obtained for the extreme 3600–3700 Å slice with

$$Q_P = 4.3 \times 10^4 , \qquad Q_D = 3.5 \times 10^4 , \qquad Q_P/Q_D = 1.24 .$$

Let us compare with the unit contrast $F_i = 5 \times 10^4$ channelled spectrum; we find

$$\frac{Q_P}{(Q_P)_{CH}} = 0.27 ; \qquad \frac{Q_D}{(Q_D)_{CH}} = 0.31 .$$

Hence, at least in this favorable case, the solar spectrum does not compare too badly with the 'optimal' one. Nature, after all, has not been so totally uncooperative as we feared. However, the *average* Q in the visible is distinctly lower, and the velocity information content of the spectrum decreases markedly in the red. Figure 8 compares the solar, Procyon, Arcturus, and Sirius quality factors.

For optimal design of any radial velocity system, and in particular of our accelerometer, it is essential to know how the quality factors will be affected by decreasing resolution. Figure 9 presents the results.

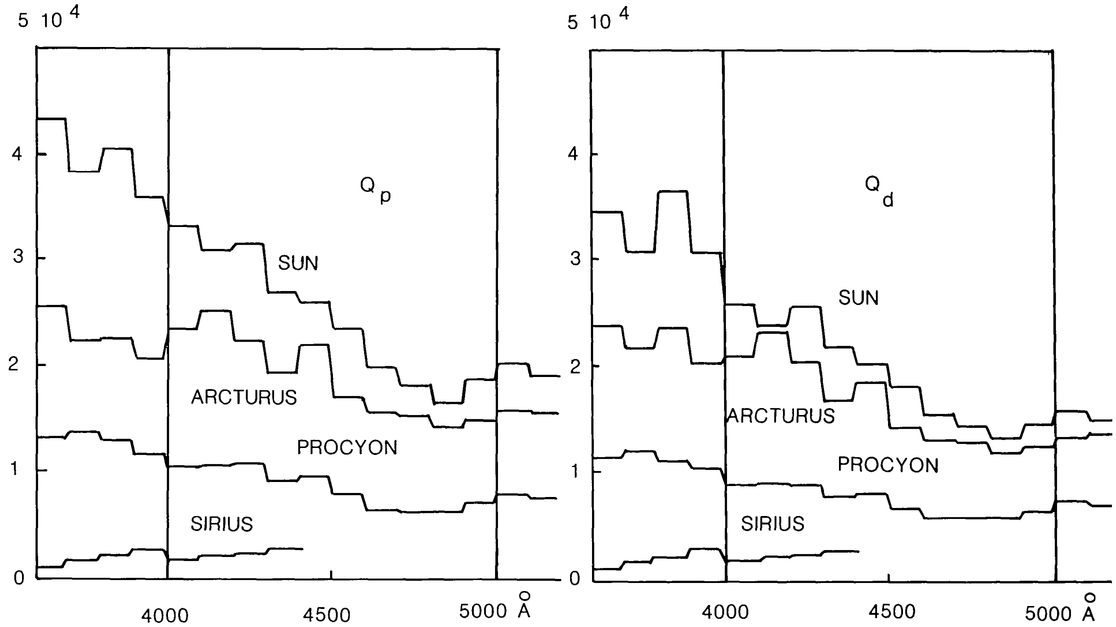

Fig. 8. Quality factors for the solar (G2), Arcturus (K2), Procyon (F5), and Sirius (A1) stellar types; the results in this last case are tentative. The factors are given here as computed from the four different Atlases, which have somewhat different resolutions; Connes (1983) gives many more curves, and in particular Q factors for *identical* resolutions. In the same paper the greater Q factors for the solar type is also explained by comparing on the same scale corresponding portions of the three Atlases: on Procyon, the lines tend to be too weak and too wide; on Arcturus, too strong and saturated. And the Sun is just about optimal.

4.2.2. *Radial Velocity Error Predictions*

Let us now study the radial velocity errors which may be computed from (11) and (13). We first consider the ideal case of a unit efficiency infinite resolving power device outside the atmosphere: in this way the theoretical (and minimal) radial velocity error will be found. Some figure has to be put in for telescope aperture and observing time; we take as standard case ϕ = 1 m and T = 1 hr. The figures for P_{AV} are interpolated for a G_2 star from Allen (1973). Then, from (9) the mean number of photons are found for 100 Å slices, and the corresponding $(\delta V_{\Sigma})_{\mathrm{RMS}}$ computed; these are next treated as independent, optimally weighted and averaged. One finds $(\delta V_{\Sigma})_{\mathrm{RMS}} = 16\ \mathrm{cm\ s^{-1}}$ for a 10th magnitude solar type star using the entire 3600–8000 Å range. Since $(\bar{N}_{\Sigma})^{1/2}$ drops by a factor of 10 for a 5 mag increase, we may plot this limit on Figure 10 as a slope 1 straight line.

Let us next consider an actual spectrometer on the ground. For atmospheric transmission we use the figures given by Allen (1973); these take into account only the continuum absorption, but this is legitimate since any molecular band will have to be carefully excluded. They are of course optimistic, since they refer to a perfect night and observations at zenith. For quantum efficiency we take the figures given by Geary (1981) for a RCA thinned backside-illuminated 512 × 320 CCD array. The choice of the resolving power R is a complex matter, which should be made objectively from a full study of all parameters involved; this has not yet been carried out. We feel that, at least

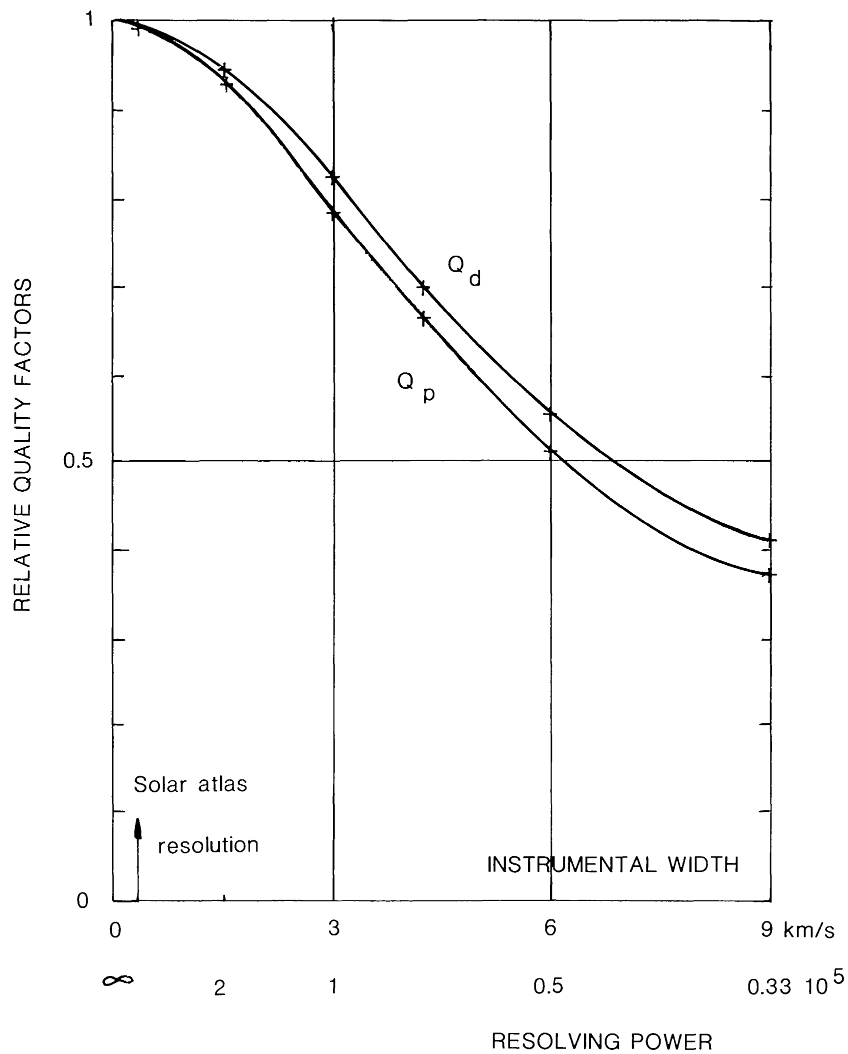

Fig. 9. Reduction of the quality factors for decreasing instrumental resolution; solar spectrum case, average result for the 3600–5200 Å range. Curves are normalized to unity at infinite resolution; a slight error arises from the actually finite resolution of the used solar atlas, i.e., 7 to 8 × 10^5 (indicated by an arrow). Two abscissae scales are provided; the first is the instrumental half-width expressed in velocities; the second is decreasing resolving power. The assumed ILS is triangular, which should describe well the proposed spectrometer, the theoretical resolution of the echelle being about 2 × 10^6 (at 3600 Å). Crosses represent actually computed figures.

for stars not too far from solar type, taking $R \simeq 10^5$ should be roughly optimum; then $(Q_P)_{10^5}/(Q_P)_\infty = 0.78$ from Figure 9. Next we take figures relevant to the spectrometer proposed by Connes (1983) and very briefly described here in Section 5.2. The overall result is

$$\delta V_{\mathrm{RMS}} = 1.1 \mathrm{~m~s^{-1}} ,$$

again for a $V = 10$ solar type star, 1 m telescope, 1 hr observing time and pure photon noise case; the spectral range is 3600–5200 Å. This prediction may be optimistic but does represent at least what the experimenter must shoot for.

For a Procyon-type star, the same spectrometer and same spectral range give 2.5 m s^{-1} and for Arcturus type 1.5 m s^{-1}. The corresponding (fully demonstrated) figure from CORAVEL is 50 m s^{-1} for the Arcturus type. Hence, an improvement of 33 times is in principle achievable. However, still in the Arcturus-type case, an extension of the range towards the red should produce some additional gain; this has not been computed so far, the spectrum not being available in numerical form. The Sirius case is slightly different. First, the numerical data were not available; hence, the Atlas tracings had to be redigitized to start with, and because of very poor operation of the used device, the Q-factors must be considered less accurate than in the other cases. Second, the available spectral range extends only to 4400 Å; hence, our tentative result, $\delta V_{\mathrm{RMS}} = 15\,\mathrm{m\,s^{-1}}$ refers only to that range, and is not directly comparable to the other figures; still the improvement to be expected from extension to 5200 Å would be moderate. A similar study will also be made for an M star in which case the red and near IR range should be far more important because of increases in Q and $\bar{N}_{\Sigma}$, simultaneously. Finally, it is also intended to study a few typical spectra of galaxies*.

The detector noise case is next considered. While a more general treatment is given by Connes (1983), we merely give here the results for a cooled CCD array; the main limitation arises from readout noise and not from residual dark current. Quoted figures generally range between 10 e^- and 100 e^- RMS equivalent noise/pixel/readout, which means here $10^2 < \bar{N}_D < 10^4$. We consider (and plot on Figure 10) a 33 e^- readout noise, and all other parameters as in the pure photon noise case. We find $\delta V_{\mathrm{RMS}} = 59$ cm s^{-1} again at $m_V = 10$, for one hour of integration. Two remarks are required: first, the representative curve is a now a slope 2 straight line, which intersects the slope 1 photon noise line at about $m_V = 11$; for all brighter stars the detector noise is small or negligible. Second, we must now make a distinction between integration time and measurement time: when searching for long period oscillations (or extrasolar planets), one hour of continuous integration may prove adequate. However, if we are looking for the so-called 5 min oscillation, the maximum sampling interval must be about 100 s, i.e., 36 times shorter; each corresponding integration requires a separate readout. Altogether, we find that for the same global 1 hr measuring time, velocity noise is increased by a factor of 6, with cross-over at about $m_V = 7.5$.

As far as the two principal programs presented here (seismology and extrasolar planets) are concerned, we are dealing with virgin fields; quite obviously, an accelerometer, once tested and proved, will have a full time job with (let us say) $m_V \leq 7$ stars. Under these conditions, detector noise from existing CCD arrays is almost negligible. Hence, we have not yet studied other possibilities, e.g., intensified arrays, or altogether different detectors, which would certainly give improved results at high magnitudes (see note added in proof).

* From our fully general demonstration one sees that, in the case of galaxies also, our method of tackling the spectrum should be optimal for radial velocities. However, from rather simple considerations one guesses that present methods are using far more of the available information than in the case of stars; consequently, the actual gain to be expected must be less.

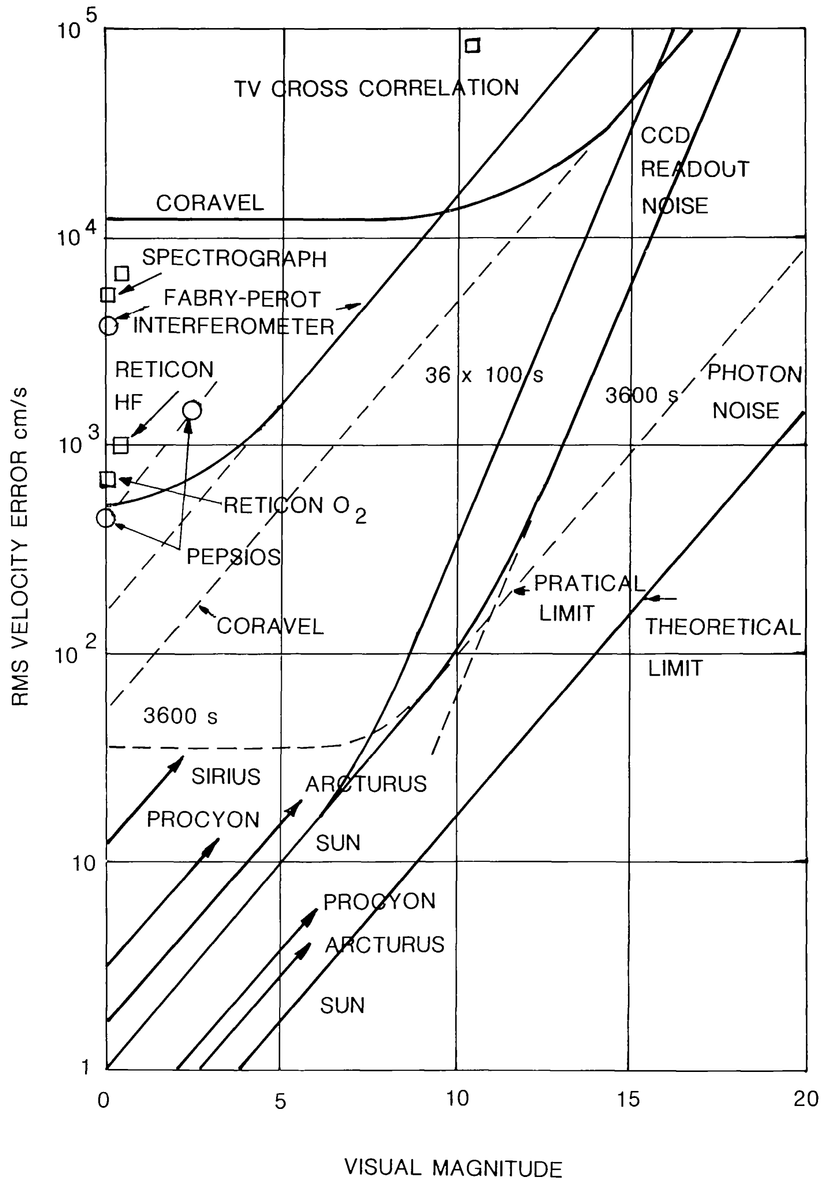

Fig. 10. Comparison of performance for various devices, as a function of magnitude. *Photon noise*, *theoretical limit*: slope 1 straight line corresponding to unit efficiency device outside the atmosphere; solar type star, 1 hr observing, 1 m telescope. Since this 'limit' has been computed from the 3600–8000 Å, $R = 7 \times 10^5$ solar atlas, it might in theory be further reduced with data from a higher resolution, wider range spectrometer, but the actual improvement would be very small. Procyon, Arcturus, Sirius: only the lower part of the straight lines is given for clarity. Due to reduced resolution and range in the corresponding data, a somewhat larger gain is in principle achievable; but this remains small on the figure scale. See Connes (1983) for discussion. *Photon noise*, *practical limit*: potential limits for a feasible accelerometer. Same stars, telescope and time, but from the ground, with CCD silicon detector, resolution $R \simeq 10^5$ and instrumental transmission 0.1. Spectral range 3600–5200 Å corresponding to Section 5.2 spectrometer. *Detector readout noise* (solar type star only): same as above, but adding 33 e^- RMS readout noise as discussed in the text. Two cases are presented: a continuous 3600 s integration or 36 successive 100 s ones each followed by a separate readout. The curves are slope 2 straight lines. The horizontal straight line at about 30 cm s^{-1} corresponds to detector saturation for 3600 s integration time. For 100 s integration, saturation takes place only for $m_V < 0$. *CORAVEL*: actual performance on faint stars from Poncet (1978), is represented by the full slope 1

5. Lasers, Spectrometer and Interferometer

We are now returning specifically to the stellar absolute accelerometer, and considering some problems of a more practical nature. Again we are mostly presenting summaries and conclusions from the fuller study given by Connes (1983).

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line at $m_V > 14$. The Arcturus spectral-type case, for which the mask is optimal, is indicated; 1 m telescope, 1 hr observing, 3600–5200 Å range, bialcali photomultiplier. One finds $\delta V_{\rm RMS} = 50$ m s^{-1} at $m_V = 10$. Horizontal line: approximate systematic errors. Dashed line: extrapolation to bright stars, supposing systematic errors eliminated. One sees that our 'practical limit' (for pure photon noise) is about 33 times less (Arcturus case; same telescope, observing time, spectral range and spectrometer transmission). The major part of this theoretically possible improvement is due to more efficient use of the velocity information given by the spectrum: optimal treatment of line profiles, strong and weak lines used, no scanning of the correlation function (*all* the required information is *simultaneously* available from the CCD). Secondary factors are: (a) higher instrumental resolution ($R = 10^5$ versus about 3×10^4) and no input slit loss, both resulting from much larger echelle plus use of image slicer. (b) Higher Q.E. due to silicon detector (this would become more important for M stars). *Fabry–Perot interferometer*: lunar and planetary laboratory device discussed in Section 1. Slope 1 straight line: computed photon count limit as given by MacMillan (1982) for solar-type star, but reduced here to a 1 m telescope, 1 hr observing. The level of systematic errors (at least as found on laboratory sources) is crudely shown by the horizontal line; it is 5 m s^{-1}, i.e., 60 times less than for CORAVEL; but the photon count error is 5 times larger than the demonstrated CORAVEL one, which means a factor of 25 increase in telescope time to get the same result. Single isolated point: actual RMS deviation recorded on Arcturus (with a somewhat primitive form of the system; Serkowski *et al.*, 1978). The given figure was 26 m s^{-1} RMS error for 1.5 m telescope, about 1 hr observing. The represented point (39 m s^{-1}) corresponds to a 1 m telescope. The ratio compared to CORAVEL is now 78, corresponding to 6100 in recording time. *PEPSIOS*: results from a search for stellar oscillations carried out with a PEPSIOS multiple FP system on 9 bright stars by Traub *et al.* (1978). The bandpass is simply set on one flank of the 6678 Å Fe-line; one measures the intensity fluctuations and computes the Fourier transform. The RMS velocity error is estimated from the measured photon count. Only two points are presented here, corresponding to Arcturus and ε Cyg. They are reduced to 1 hr observing time, 1 m telescope. Next are plotted as isolated points (small squares) results for which observers unfortunately do not give the data (e.g., counting rate and/or observing time) necessary to unravel photon noise from systematic errors. Hence, no reduction to 1 m telescope, 1 hr observing has been attempted (if it were done, all results would degrade appreciably). In particular, the apparent proximity of the *spectrograph* points to the *interferometer* one is grossly unfair to the interferometer. *TV cross-correlation*: single point tentatively representing the results of an interesting determination of RV by an original technique (Da Costa *et al.*, 1977): numerical cross-correlation of the unknown velocity star spectrum (recorded with a SEC vidicon at the output of a Coudé spectrograph) with that of a similar type known velocity star. Spectral range 115 Å near 5180 Å; 185 cm telescope, exposures 10 to 40 min. Errors of the order of 1 km s^{-1} are quoted for stars in the $m_V = 10$ to 11 range. *RETCION*–O_2: from Smith (1983). The Coudé spectrograph of the 2.7 m MacDonald telescope has been fitted with a RETICON detector; a set of up to 9 lines of Arcturus and Aldebaran near 6300 Å has been observed during 4 nights, taking 7 telluric O_2 lines as standard. An overall error of 7 m s^{-1} is quoted. *RETCION–HF*; from Campbell (1983). Similar technique, with the 3.5 m CFH telescope but using an HF cell near 8700 Å. The RMS scatter of velocities from 3 nights devoted to Procyon is about 10 m s^{-1}. In both cases, the original papers should be consulted for exact meanings of these figures and discussion of error causes. They are plotted here merely to provide the order of magnitude for the most accurate present results. *Spectrograph*: best published results in a classical determination of radial velocity using photographic plates, from Griffin and Griffin (1973). The figures given are 50 m s^{-1} single plate standard deviation for Arcturus and 60 m s^{-1} for Procyon. This was achieved with a 2.5 m telescope and exposures ranging from 1 to 6 hr.

5.1. Lasers and Beat Detection

The maximum velocity change is now ± 30 km s^{-1}, in the worst case of an ecliptic star observed at both quadratures. Still considering the He–Ne laser, with $N_0 = 4.74 \times 10^{14}$ Hz, the total required tuning range is now 94 GHz, about 100 times too large, while the maximum beat frequency is 47 GHz. With IR lasers the absolute frequency range is reduced; for instance with the methane stabilized He–Ne laser at $\lambda_L = 3.39\,\mu$, $B_{N(\max)} = 17$ GHz and with the saturated fluorescence CO_2 laser at $\lambda_L = 10.2\,\mu$, $B_{N(\max)} = 5.9$ GHz. However, the optical problems inherent in the use of these lasers are not trivial; furthermore, only the beat detection difficulty is alleviated, not the tuning one since such lasers have roughly the same *relative* tuning range, because relative Doppler linewidths are about the same. Hence, the problem is distinctly more arduous than in the solar case; nevertheless, it is already solved thanks to spectacular recent advances in the field.

Let us quote from just a few of many recent papers. Burghardt *et al.* (1979) have produced beats at up to 80 GHz between a 6328 Å, 0.25 mW stabilized laser, and a 2 mW tunable dye laser; the mixed beams were focused on a Ga–As Schottky diode, operated in avalanche mode and mounted within a millimeter waveguide; the microwave radiation generated by the diode was detected by standard techniques, the beat signal being 15 dB above noise for an integration time of 1 m s.

Then, Daniel and Steiner (1981) used an argon ion laser at 5146 Å and a tunable sodium dimer ring laser; the two beams were focused on a MIM (metal-insulator-metal) point contact diode, acting as a triple mixer since it also received a microwave beam at about the expected frequency difference (up to 122 GHz). Low-frequency beat signals were ultimately detected.

Lastly, Evenson (1981) reports on an essentially similar experiment between two visible dye lasers in which the microwave generator was replaced by a far infrared laser; the detected frequency difference went up to 2.5 THz, while the predicted limit for the technique is 30 THz. This is far more than what we need.

What about the limitations of tunable lasers? Helmcke *et al.* (1982) describe a single mode CW dye laser, which is actually frequency stabilized by reference to an external passive cavity (i.e., a FP etalon), plus a 6328 Å laser. A 1 kHz linewidth has been achieved, together with 50 mW output power. A continuous scanning range of 3 GHz is available by simultaneously tilting two internal mode-suppressing etalons and retuning the cavity. When it is exceeded the system may be reset, in principle anywhere from near UV to near IR.

While these systems are admittedly quite complex, the technology is presently progressing fast, due to considerable metrological interest. Altogether, it is clear that the problem of beat detection, while delicate, will not ultimately set a limit to the performance of stellar accelerometry. However, it is desirable to use simpler solutions at least to start with. In particular, how much actual work can we do by taking merely the 6328 Å He–Ne laser that will be adequate (both for the stabilized and the tunable units) with the solar accelerometer? The most obvious solution is to restrict sky coverage, or observing windows or both. We consider first the worst case, that of ecliptic stars. If observed close to opposition, the beat frequency change due to Earth orbital motion is about 800 MHz in 24 hr, but at quadrature we may use 14 consecutive nights without exceeding the allowed 900 MHz tuning range; however, in the first case we have full nights and in the second, very short ones. As latitude increases the allowed periods lengthen, and for two small zones around the ecliptic poles, such restrictions almost vanish. In this way we may test the whole method under the simplest conditions, as far as lasers and beat detection are concerned; but for a regular observing program, we need something else.

Two different but compatible procedures may be used to restrict beat frequency range. The first involves switching the tunable laser to the next FP order when the so far used one has reached the edge of the permitted tuning range. The full discussion of FP spacing choice is given by Connes (1983); starting from a $R = 10^5$ spectrometer resolution, we find that the largest practical FP thickness is $t = 1.6$ cm which gives 9.4 GHz for the frequency interval between orders. This is 10 times less than the full range required for an ecliptic star, but still 10 times more than we want. Hence, this technique by itself is not sufficient.

The second procedure involves applying a change of FP spacing; there is no restriction due to the finite FP free spectral range, and this change may be arbitrary small, i.e., tailored to keep the laser frequency within given bounds. However, a change of philosophy is also implied: the two stretching factors and the two velocities are no longer kept equal. Still, their differences are precisely known: one simply compares the two spectra (before and after the switch) and the corresponding beat frequency change is measured. Hence, stellar accelerations now require a calibration; however, this operation is performed directly in terms of frequencies and does not involve the stellar spectrum, only that of the FP, for which the SNR is much higher. The operation is repeated up to a maximum of 10 times; then, one reverts to the 'normal' FP spacing and makes use of the first procedure, i.e., of a different FP order. Unquestionably, accelerometry performed in this way loses some of its conceptual simplicity, and one should fear increased systematic errors, the magnitude of which only experiment will show; but the equipment needed is very simple. The ultimate goal must remain full-frequency range beat detection.

5.2. Spectrometer and Optical System

The spectrometer and associated optics briefly discussed here are merely a modified version of the system proposed by Connes (1978). The main difference is that the mask must be replaced by a CCD array which is much smaller. Also the various mechanical devices proposed for accurate measurement of the velocity shift are no longer required.

One essential feature remains common to both projects: the input scrambling fiber. The full description and the results of the laboratory tests will not be repeated here, only the main conclusions. The purpose of the device is the same: any comparison of stellar and reference spectrum is highly sensitive to seeing and guiding errors which affect only the first, because of changing illumination on the input slit, and also to a lesser degree on the grating. In the present system we have demonstrated the result to be largely independent of all ILS changes, but the proof requires this ILS to be exactly the same for the FP and stellar spectra. Hence, both beams will be sent through the same fiber. The tests reported by Connes (1978) detected no measurable departure from perfect scrambling, i.e., no output beam change when a point source was scanned across the fiber input. However, the overall accuracy sought for in the new proposal is much higher; clearly, more accurate tests should be performed, and with more recent fibers, the technology of which has been fast improving. In any case, we notice that the proposed scrambler is fully compatible (without additional loss of energy) with image rotators, as done for instance by Connes (1966) and Serkowski (1979). These would insure that any residual systematic difference between stellar and FP beams would exhibit perfect average circular symmetry. One last point: the fiber would also be used as a convenient mean of taking the light from the telescope prime focus to a temperature controlled laboratory, which would house spectrometer, lasers, white light source and interferometer; then an auxiliary fiber is needed to take the FP outgoing beam to the input of the first fiber (Figure 11).

The scrambling fiber output goes to an image slicer which plays an important (although not essential) role, by enabling the spectrometer to accept to full seeing disk

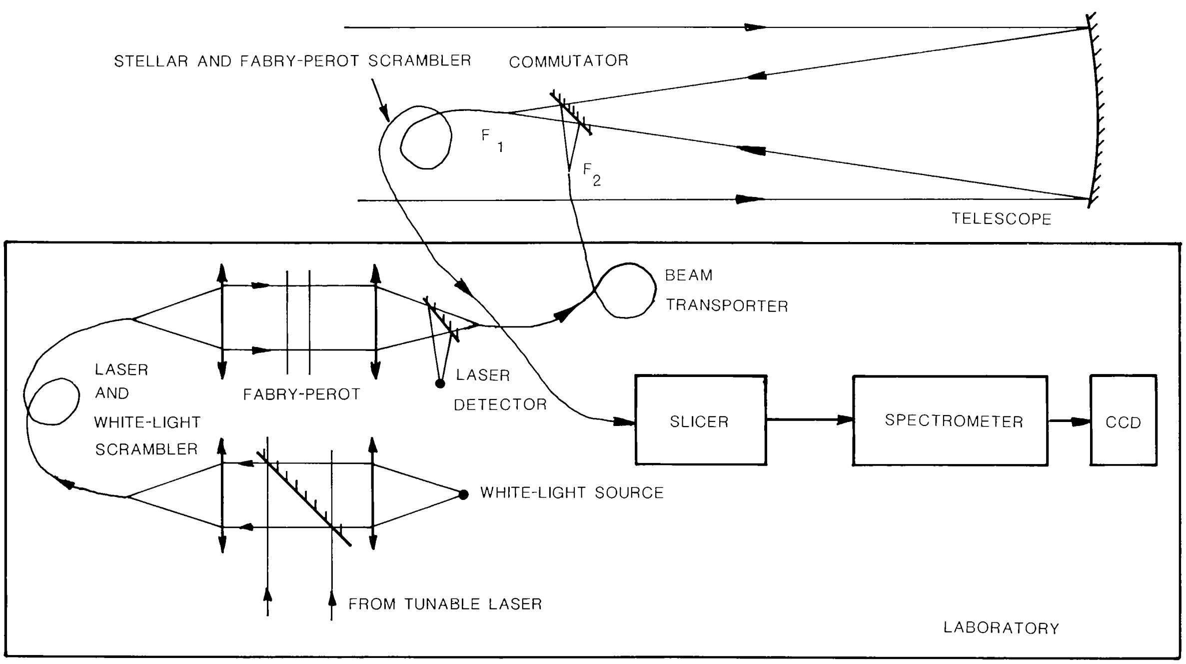

Fig. 11. Highly schematic optical diagram of stellar accelerometer. All devices are located in the laboratory, except the commutator which is close to telescope focus. Lenses could be added near F_1 and F_2 for matching beam convergence to fiber acceptance, as discussed by Connes (1978); they can be used in immersion, as done by Serkowski (1979). For even better scrambling, each fiber may be replaced by a conjugate pair, i.e., a set of two successive identical ones with a coupling lens in between: the output of the first and the input of the second are located in the front and rear focal planes, respectively; each 'sees' the other at infinity. Also image rotators may be added easily at scramblers input or output or both; only experience will show whether such a degree of elaboration is required. The 'beam transporter' is actually a similar fiber, but does not have to provide additional scrambling.

at the wanted resolution. One novel form of slicer was proposed, but not tested by Connes (1978); it could be used here, but is not required: other more familiar forms of slicers appear adequate. The often voiced objections against slicers when accurate line positions are concerned, do not apply here: first, the illumination at slicer input will be totally stable; second, any residual effects will be the same for the FP and stellar beams. A very high slicing ratio is not required: we find that a ratio of 4 is adequate to feed the proposed spectrometer from a 1 m telescope, working at $R = 10^5$, with no loss at up to 5 arc sec seeing disk diameter.

The spectrometer itself may be understood as a much enlarged version of CORAVEL; high transmission is also a main goal, but it is combined here with about three times greater resolution. The main mirror is spherical and centered on the echelle; this eliminates coma and astigmatism, while the f ratio is taken high enough to make spherical aberration negligible. Good efficiency results from the use of the echelle in Littrow mounting and of a prism for providing the cross dispersion; this not only reduces losses compared to a grating, but gives nearly equidistant echelle orders, hence, a more efficient use of the CCD. The proposed system makes use of 204 × 408 Bausch and Lomb echelle, with 63° blaze angle and 316 grooves cm^{-1}, combined with a 800 × 800 pixels Texas Instruments CCD array; pixel size is 15 × 15 μ and array size 12 × 12 mm. Echelle free spectral range is 177 cm^{-1}; we use 48 orders to cover the 3600–5200 Å spectral range at a resolution of 10^5; as already stressed, these choices are somewhat arbitrary, but should very roughly correspond to an optimum. For spectrometer efficiency, we take the same figure as for CORAVEL, i.e., 0.25, and for the rest of the system 0.4, leading to overall 0.1 transmission; corresponding figures have been used in our discussion of fundamental velocity errors (Figure 10).

The increased size compared to CORAVEL introduces one difference: a direct vision prism would be difficult to make, and inefficient due to excessive thickness of the required high-dispersion material. A simple prism introduces slight curvature in the echelle orders, which makes merely adressing of the pixels more complex. As discussed by Connes (1983), a water or CCl_4 prism is proposed, which provides excellent transmission throughout the spectral range; this unorthodox solution is a consequence of the very good results long recorded with an almost similar size ethyl cinnamate prism built by Couder (1933). Also, the small size of the array compared to that of the mask requires at the output a high aperture condensing system; a preliminary study of a plane mirror folded Schmidt has been carried out by Lemaitre at the Observatoire de Marseille. Other detectors with larger pixels (for instance the RCA 512 × 320 array, with 30 μ pixels), make the design easier. Since there are fewer pixels, it is necessary (and feasible) to use several of them in parallel if one wants to keep the same resolution and spectral coverage*.

* At the *December 1983 Florence Astronomy Meeting*, a new CCD under development for the Anglo-Australian telescope has been described by Mackay. This would improve the results plotted on our Figure 10 on several counts. The larger pixels (26 × 26 μ) indeed make design of the Schmidt camera easier. The 1500 × 1500 pixels array permits an increase in spectral range or resolution or both, which means a slight reduction of the overall δV_{RMS} in the photon noise practical case. From the 6 e^- readout noise a distinctly larger improvement would be realized in the detector noise case.

5.3. Interferometer

The Fabry–Perot interferometer must have low finesse and be usable within a broad spectral range. This is best achieved with Al or Ag coatings; the increased absorption compared to dielectric layers is irrelevant. The effect of wavelength-dependent reflectivity and phase shift are treated by Connes (1983), and shown to be negligible: there is no requirement anywhere that the interferometer should exhibit strictly constant free spectral range and finesse. Actually, a Michelson interferometer (which does provide a constant finesse of 2 at unit contrast) could be used, while the FP may only give low finesse at reduced contrast. Since the SNR in the channelled spectrum analysis is not expected to be a dominant factor, the slight additional complexity involved does not seem warranted.

The interferometer tuning range, just like that of the laser, must be $\pm 10^{-4}$ in order to follow an ecliptic star throughout the year. Unlike in the solar accelerometer case, this cannot be done by pressure variation, because gas index dispersion would introduce an unacceptable mismatch between the spectra and their mask functions; the problem is the same as that treated by Connes (1978). Hence, tuning must be purely geometrical. Variation of incidence, while theoretically possible, would almost certainly reintroduce systematic errors with the same periodicities as the principal Earth motions, i.e., 24 hr and 365 days. These might prove acceptable but a much better solution is to keep the incidence constant (and near-normal) and vary the spacing. Many versions of tunable FP interferometers, some of them commercial, appear suitable: the magnitude of the spacing change required is easily within range of piezoelectric drives.

The role of interferometer plate errors and imperfect parallelism may be discussed, but the actual magnitude of the resultant systematic errors cannot be predicted without specific tests. While the desired finesse is very low, this does not mean that the required accuracy, in flatness of surfaces or parallelism, is easy to achieve. The problem is a complex one: these various errors mean that instead of having at a given epoch one well-defined FP spacing T, we have in fact a continuous range of spacings ΔT. Since T disappears totally from the final result (Equation (1)), there is simply no direct effect. However, hidden behind this simple demonstration, is the assumption that the FP illumination from the white light source and the tunable laser beam is the same. One easily sees that any possible error vanishes if either of three conditions is satisfied: (a) the etalon is perfect; (b) the illumination is non-uniform and different for both beams but stable; (c) the illumination is unstable but identical. Any residual velocity error is a third-order effect which has to be compounded from terms involving etalon errors, changing illumination, and systematic differences between the two beams *at the same time*. They are obviously difficult to estimate *a priori*. Just as in the spectrometer case, we have to make illumination as stable and identical as possible; again this will be best achieved with a second scrambler which is introduced at the FP input within the path of the laser and white light beams (Figure 11). However, unlike in the spectrometer case, perfectly identical scrambling may no be achieved for the two beams, because they have very different spectral contents. Here again, one may have to add image rotators as an additional precaution.

An attractive possibility lies in the use of a spherical Fabry–Perot etalon (Connes, 1958) which appears capable of reducing the effect of surface errors and misadjustment simultaneously. Altogether, it is not possible without specific tests to quote any figure for the ultimate, incompressible, *systematic* errors we shall meet in stellar accelerometry.

6. Discussion: The Ultimate Possibilities of Stellar Accelerometry

Essentially, our device will deliver astronomical accelerations. What are the basic accuracy limitations for our intended programs? And (broadening somewhat the outlook), can we dream about altogether different applications? At a time when more and more interest focuses on the unseen Universe, which may become sensible only through the pull exerted upon the visible one, if we are offered a novel method, potentially far more sensitive than any wielded so far, the question is at least worth asking. Let us suppose we have long played the game right, and airily dismiss even the memory of systematic errors and detector noise. We have reached the time when only photons count. As, indeed, some day, we shall. What are the fundamental errors for *periodic* accelerations?

We start from the RMS velocity error δV_{RMS} for 1 hr observing time, which we take as standard case. Let us suppose we actually make M observations at M equidistant epochs; each lasts 1 hr$/M$ and the interval between epochs is adequate to sample correctly the expected waveforms. Then the resulting RMS error on the velocity amplitude of any periodic term is $q\delta V_{\text{RMS}}$, where q is a numerical factor of the order of unity, depending on the exact technique (e.g., on whether apodization is used or not). Hence, we may take our computed δV_{RMS} as giving approximately the minimum detectable velocity amplitude in 1 hr actually spent on the sky; for a program involving p hours altogether, this figure would be reduced by $\sqrt{p}$.

If the period of the looked for oscillation is P, the corresponding error on the linear amplitude is $\delta x_{\text{RMS}} = (P/2\pi)\delta V_{\text{RMS}}$ and on the acceleration amplitude $\delta\gamma_{\text{RMS}} = (2\pi/P)\delta V_{\text{RMS}}$. Let us consider our 'practical' case and a $m_v = 10$ solar-type star; then $\delta V_{\text{RMS}} \simeq 1$ m s^{-1}. If we study a $P = 5$ min oscillation, then $\delta x_{\text{RMS}} \simeq 50$ m and $\delta\gamma_{\text{RMS}} = 2$ cm s^{-2}. If the system were to be set up at the South Pole and observe for 120 hr, these figures would be reduced by a factor 11 and the minimum detectable velocity would be about 10 cm s^{-1} (and the amplitude 5 m). This must be compared with the actual 10 cm s^{-1} velocity noise found in the 5 min solar oscillation region by Fossat at the Pole (Grec *et al.*, 1980), under exactly these conditions. In other words, if photon noise were to be the only limitation, we should be able with *a* 1 m *telescope to get results equivalent to the South Pole ones on a* $m_V = 10$ *solar type star.*

If we now look for a $P = 10$ yr $= 3.5 \times 10^8$ s planetary perturbation (still with $\delta V_{\text{RMS}} = 10$ cm s^{-1} on a $m_V = 10$ star, with $T = 1$ hr), then $\delta x_{\text{RMS}} = 5 \times 10^4$ km while $\delta\gamma_{\text{RMS}} = 2 \times 10^{-6}$ cm s^{-2}. If our telescope manages to get 1500 hr yr^{-1} of observing, and divides its attention between 150 likely candidates, each will have been watched for 100 hr after a 10 yr program; the resulting final photon count errors are $\delta V_{\text{RMS}} = 10$ cm s^{-1}, $\delta x_{\text{RMS}} = 5 \times 10^3$ km, and $\delta\gamma_{\text{RMS}} = 2 \times 10^{-7}$ cm s^{-2}, which must be compared with the expected 13 m s^{-1} peak velocity for a Sun–Jupiter association. Of course, whether actual planetary perturbations may ever be unraveled from intrinsic stellar phenomena at a comparable level of sensitivity is an altogether different matter. Lastly, we consider $P \gg 10$ yr, i.e., the case of long-period binaries, but still within the same program aimed at detecting planetary systems; we have again data for 100 hr star^{-1}, more or less evently spread over 10 yr. The acceleration to be measured is now for all practical purposes constant. The minimum detectable one is now about 6×10^{-8} cm s^{-2}, still on a $m_V = 10$ star. Let us now investigate a solar system acceleration (Bentley's problem), produced by some dark mass in our neighbourhood. We may average results from all the 150 stars on watch; supposing them all of tenth magnitude and evenly distributed on the sky, we may now detect 10^{-8} cm s^{-2}, in any direction. This figure again takes only photon noise into account, and we do not know what the errors arising from slow changes in the atmospheres of the used stars will be; clearly, they will be reduced by averaging, just like the photon count errors.

It is illustrative to quote at this point the lowest gravitational accelerations we have been able to detect from the TRIAD (1974) drag-free satellite: the system had been designed so that all non-gravitational forces were to be reduced to about 10^{-8} cm s^{-2}; however, due to a loss of telemetry preventing in-orbit final adjustment, a residual self-bias of about 3×10^{-6} cm s^{-2} was actually observed. TRIAD was an Earth-orbiter, but nothing prevents us from making a deep space version. Hence, we may, in principle, detect weak accelerations of the same order of magnitude using as a probe either (a) a free falling mass within our own solar system or (b) the whole system itself, relative to a pack of arbitrarily chosen stars. However, these have to be in the neighborhood; for more distant ones, sensitivity decreases rapidly.

Pulsars have been proposed as suitable anchoring posts in the same problem of detecting solar system accelerations (Harrison, 1977). Their own limitation is the intrinsic slowdown which must be empirically determined for each of them and mimics a steady acceleration γ; if P and $\partial P/\partial t$ are the period and period derivative, then $\gamma = (c/P)\, \partial P/\partial t$. So far the 'best' pulsar for the purpose seems to be PSR 1952/ + 29 with $P = 0.42$ s, $\partial P/\partial t = 2 \times 10^{-18}$, $\gamma = 1.4 \times 10^{-7}$ cm s^{-2} (Gullahorn, 1978). So far also, the new 'millisecond' pulsar PSR 1937/ + 21, with $P = 1.55$ ms, $\partial P/\partial t = 1.2 \times 10^{-19}$, hence, $\gamma = 2.3 \times 10^{-6}$ cm s^{-2} is not as 'good', but in this case the figure for $\partial P/\partial t$ may be only an upper limit (Ashworth, 1983).

How do stars compare with pulsars in this context? Once we have developped an ideal stellar accelerometer, which operates by locking to the stellar lines and gauging their fluctuations in terms of terrestrial frequencies, we may thinks of our stars as clocks and question their stability. For pulsars, we have the slowdown plus various timing irregularities (the so-called 'activity'). For stars, first the good news: any truly secular term analogous to pulsar slowdown is ruled out. If things were not so, then why should integrated disk solar wavelengths happen to match reasonably well the terrestrial ones at the present epoch? The (already stressed) bad news are that we have no idea so far of how large the fluctuations with pseudo-periods of a few years will be. Altogether, and given some luck, we might find that stars are not such bad clocks after all, once we have learned to tackle them right; anyway, being *atomic* clocks, they are bound to beat pulsars (mere mechanical toys) in the long time range. But clock accuracy of course does not tell the whole story: pulsars offer a considerably more distant frame of reference; provided we find enough 'good' ones spread over the sky, they might answer very different questions from the ones we can ask our neighbour stars. Merely an example: our present proposal cannot lead to a direct measurement of galactic rotation, because accelerations of nearby stars are about the same.

A more fashionable problem is that of gravitational waves detection. Conceptually, it has nothing to do with those we have just discussed; but accurate accelerometry might detect GW in the same way and during the same programs, at least in principle. We may consider GW acting either on the solar system (hence, moving the observer), or on the distant star being watched, or both; in all cases an apparent relative acceleration would be induced. Two cases have to be considered separately. First, we have hypothetical ultra-low frequency GW, meaning phenomena with periods greater than a few years. Then, for all practical purposes, the situation is the same we discussed above: one would be looking for a steady-state acceleration between solar system and target star. Indeed, pulsar timing has already been used a tool in this context, and upper limits for ULF GW densities have been derived, using about 10 yr of accumulated observations (Detweiler, 1979; Mashloon, 1982; Bertotti *et al.*, 1983). The same limitations again apply, both for pulsars and for stars.

Far more exciting would be a search for GW with frequencies roughly comparable to those of stellar oscillations, since there are a number of sources which might presumably act as emitters of GW in this range. Is there any hope at all of detecting short period GW coming from just any place in the Universe, and merely crossing the light path? Let us ask first a plain (but somewhat naive) question: What is the smallest *periodic relative variation of stellar distance* $\delta x/x$ that stellar accelerometry will be able to detect? We have just found $\delta x_{\mathrm{RMS}} = (P/2\pi)\delta V_{\mathrm{RMS}}$ and for $T = 1$ hr on a $m_V = 10$ star, $\delta V_{\mathrm{RMS}} = 1$ m s^{-1}; supposing, for instance, $P = 100$ s, then $\delta x_{\mathrm{RMS}} = 16$ m. Adding the assumption that the star has the same absolute magnitude as the Sun, then $x = 100$ pc $= 3 \times 10^{20}$ cm and $\delta x_{\mathrm{RMS}}/x = 5 \times 10^{-18}$: with 100 hr of integration, this drops to 5×10^{-19}; for a *given absolute magnitude* of the star used as probe, this is actually independent of distance (as long as detector noise is negligible).

Hence, it is clear that we shall some day be able to measure a periodic relative stellar distance change (or in other terms, strain of the Earth-star baseline) with a sensitivity in the same league as the best proposed GW techniques. Unfortunately, this result by itself in no way implies any capability of detecting periodic GW radiation: the GW-induced computed Doppler shift from any space probe does not increase with distance x when $x \gg \lambda$ (Eastabrook, 1975; Kaufman, 1970), and the above quoted figure of 5×10^{-19} is not relevant to the problem. For all we known, it is relevant to no physical problem at all, essentially because c is finite; it is fun to quote just the same.

We are then left with the much less attractive possibility of chance detection of some low probability situation. Suppose a strong GW emitter happens to be in the immediate solar neighbourhood as recently proposed for Geminga (Walgate, 1983). Then it would induce solar and terrestrial accelerations; but our technique could not compete with the tracking of space probes, not to speak of future interferometers in space, which both make use of coherent detection. However, if the hypothetical GW source lies in the vicinity of some observed star in our program, then stellar accelerometry would be the only specific tool. Still, in order to get an unambiguous demonstration, we would have to find the *same* line in the seismic spectra of *two* stars, both close to the GW source, which may be asking for too much, as far as probabilities go.

Altogether we must with deep regret refrain from any touting of our proposal based on the hope of GW detection. Which is a pity; as a selling device, this would have ranked an honourable second best to the proof of the existence of God, which we also had to drop in Section 1, due to various advances in astronomy (not to speak of more questionable ones in philosophy) over the last three centuries.

The present paper is solely concerned with methodology. No attempt will be made to propose actual seismology or planetary search programs. Neither shall we analyse the various astrophysical limitations. In the planetary search case, we have simply no idea of how severe these will be. The successive NASA Workshops on the question have been inconclusive on this essential point, since even in the case of the Sun we do not have accurate integrated light velocities over a long enough period. The seismology situation is totally different: there is no doubt that, at least for solar-type stars, no intrinsic limitations apply to the level of accuracy demonstrated by the South Pole and Teneriffe–Hawaii seismic spectra (for presently predicted stellar oscillations, see Christensen-Dalsgaard, 1984). This guarantees a bountiful crop of scientific results in the short-time range. This essential fact may provide the goad to develop the specific instrumentation required, which happens to be the same in both cases. Hence, we may hope to see those hypothetical and long talked about planetary systems carried upon the shoulders of down to earth stellar seismology. Stellar tremors should provide the daily excitement needed to keep a slow and hazardous program alive; and we may wake-up some day and find new planets as an half-expected bonus.

Still, it is possible to say something about stellar limitations to planetary detection as they will be handled by our proposed accelerometer. This fully preserves the capability of *local spectrum checking*, hence, remains far more adaptable than either the single detector Fellgett correlation meter, or than any device making use of just one or a few lines: while we must operate the FP loop from an average velocity change defined over a wide spectral range, it does not follow that this wholesale and unadorned average will be responsible for the ultimate results. Let us first suppose that the star rythmically breathes in and out its own atmosphere in an homogeneous manner; against such perverse behaviour there is nothing we can do, except watch for a very long time and hope this oscillation will prove of low coherence, unlike planetary action. Next, we drop the central assumption made in this paper so far, i.e., that of a single, well-defined Doppler shift for the whole spectrum. All the information required to reconstruct individual line profiles and positions relative to the average, remains available, within limits set by resolution and noise*. We may, for instance, plot line bisectors, as done by Dravins *et al.* (1981) for many solar lines. These would probably be best for understanding the physical processes involved, and indeed we are used to thinking in terms of such profiles. However, they are only indirectly relevant to the business in hand; for the present purpose, the most convenient quantity will be the *spectral velocity perturbation* $\delta V_n'$ defined again by Equation (6), but extended now to fractional spectral ranges $\Sigma' < \Sigma$. While the servo operation cancels δV_n as explained, the $\delta V_n'$ term is *not* cancelled and does represent intrinsic stellar effects irrespective of possible planetary action. By playing with the limits of Σ' and with the weighting function $F(\sigma)$, we may compute $\delta V_n'$ either for small or large ranges, or for groups of lines selected according to some criterion, or just for the line tips, or bases, etc. Of course, we have an endless variety of possible situations, and no clear cut demonstration can be presented. Still, it is obvious that a system such as ours which records *all* the spectral information at *all* epochs, leaves open the possibility of treating it optimally at program end, whatever happened in between. In other words, *a posteriori* corrections deduced from internal evidence will be feasible; we shall learn as we go. This is a very important feature at the onset of a somewhat uncomfortably long-term program considering our present ignorance. Here are but two simple examples: we shall be able to discover and take into account non-fully dark companions on the way; and we may conclude that some classes of lines (e.g., those formed above the convective region) are intrinsically more stable than others, and should preferentially be used. Unavoidably, any conclusion based on a restricted spectral range will have lower SNR than predicted by Figure 10.

7. Conclusions

No actual testing program, estimation of costs, etc., will be presented here; as in any exploration, this cannot be meaningfully done until at least a base camp has been established, and none is presently available. Still, two general remarks, both pertaining to the development of the instrumentation must be made at this point. The first is that two fully independent ideas have actually been presented. The title and accent have been on absolute accelerometry; but the photon-noise limited measurement of radial velocities with a grating-spectrometer detector-array computer system is a separate concept, which may and should be tried separately. No interferometer nor laser is required; the curves (Figure 10) show that indeed for sufficiently faint stars (say $m_V > 12$), these will never be needed at all because the photon noise level will be too high, and perfectly standard spectrometer calibration procedures are fully adequate. Of course, no seismology nor planetary searching is then conceivable and we consider more classical

* The so far quoted 10^5 resolving power figure roughly corresponds to a broad maximum of the overall SNR, when fundamental noise sources are solely considered. When the astrophysical factors presently discussed are taken into account, it may turn out that a distinctly higher R will lead to better overall results. Roughly speaking again, figures from 10^5 to 5×10^5 appear achievable; these would provide fairly good sampling of line profiles. The price to be paid is: (1) a decreasing fraction of the seeing disk may be used (this is partly compensated by an increase in the Q factors); (2) an increase in cost, since several detectors in parallel (or future detectors with more pixels) are required to cover the same spectral range.

radial velocity programs. Perhaps optimistically, it seems that construction and operation of that reduced system should not prove unduly difficult; partial testing of the basic ideas might indeed be achieved with existing equipment. However, correct operation of the complete accelerometer (including FP and lasers) will be an altogether different kettle of fish; one must visualize the residual systematic errors as being slowly understood and reduced as the years go by.

The second remark relates to the even longer term outlook. While we see from the curves (Figure 10) that in principle seismology should become practical for a fair number of stars, we must remember that this is a highly time consuming activity: on the Sun, the best seismic spectra are the five-continuous-days South-Pole one (Grec *et al.*, 1980) and the eighty quasi-continuous days one from two stations (Claverie *et al.*, 1984). It is impossible to hope for anything even remotely approaching that degree of coverage on stars, unless several telescopes are dedicated to the program. Fortunately, if we consider the specific requirements of accelerometry on relatively bright stars, we may confidently assert that a suitable telescope could be built for a small fraction of the usual cost; possibly it would be cheaper than the accelerometer itself. This assertion is based on our experience in developping the Meudon IR collector (Chevillard *et al.*, 1977). Without attempting a description, we may at least give here the outline of a proposed solution. The telescope would consist of just one parabolic mirror in the 1 m class, giving perhaps 2 arc sec images and directly feeding the light into the fiber-scrambler at prime focus (there is no point in using mosaic mirrors, as done at Meudon, for such small sizes). The fiber input, plus some auxiliary fibers for guiding and finding, would be held by an extremely light tripod structure. A compact and low precision altazimuth mount (without accurate driving gears), specifically designed for servo guidance would be adequate. Wind torques would be low, and taken care of anyway by the servo system plus the scrambler. Hence, no dome is necessary. The builder would actually implement that seldom realized optician's dream: putting most of his cash into the light collecting mirror, and doing away with all the frills. The undertaking is vastly simpler than that demonstrated at Meudon, and less hazardous than the construction and fully sucessful operation of the accelerometer itself.

Such is not, of course, the way to start the program; mere laboratory demonstration of the accelerometer shall prove at best a lengthy and tricky business. Next, the scrambler may easily be adapted to any small telescope (no Coudé focus needed). However, it must be clear from the onset that one or several low accuracy dedicated telescopes will ultimately be required for probing stellar insides. Since the only other proposed seismology tools are stellar photometric satellites (Praderie *et al.*, 1984), that may still be a small price to pay. Furthermore, radial velocities are likely to prove a better tool than brightness fluctuations, except for fast rotating stars (Fossat, 1984).

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While I do not remember debating radial velocities with P. Fellgett, his direct influence (here as in other matters) is clear cut. Those able to read between lines will also recognize a distant offspring of the powerful and elegant optical lever principle developed by R. V. Jones (1959): stellar accelerometry is ultimately feasible because stars have been designed with such built-in levers, in frequency space; we only need to grasp the proffered handles. The responsibility of P. Jacquinot, while less manifest, has been no less real: his students all learned the trick of treating lowly instrumental problems from fundamental viewpoints.

The French astronomical authorities of INAG and the Observatoire de Meudon provided – by withdrawing all my research facilities – that blessed peace of mind most convenient for evolving new concepts; not, however, for testing any part of them.

Note added in proof. The most suitable photon-counting detector for our stellar interferometer should be a specifically modified version of the PAPA detector (Papalios and Mertz, SPIE 'Instrumentation in Astronomy IV', p. 360, 1982). The required Gray-binary coded masks would be built starting from an *in situ* photograph of the FP spectrum as given by the échelle and prism. The resulting device would be relatively inexpensive, and the connexion to the computer particularly simple (much more so than with the CCD). Also the need for a Schmidt camera disappears. The device would not be suitable for bright stars, because of counting rate limitations.

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