HIGH-RESOLUTION BROADBAND SPECTROSCOPY USING AN EXTERNALLY DISPERSED INTERFEROMETER

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ABSTRACT

An externally dispersed interferometer (EDI) is a series combination of a fixed delay interferometer and an external grating spectrograph. We describe how the EDI can boost the effective resolving power of an echelle or linear grating spectrograph by a factor of 2–3 over the spectrograph’s full bandwidth. The interferometer produces spectral fringes over the entire spectrograph’s bandwidth. The fringes heterodyne with spectral features to provide a low spatial frequency moiré pattern. The heterodyning is numerically reversed to recover highly detailed spectral information unattainable by the spectrograph alone. We demonstrate resolution boosting for stellar and solar measurements of two-dimensional echelle and linear grating spectra. An effective spectral resolution of ~100,000 has been obtained from the ~50,000 resolution Lick Observatory two-dimensional echelle spectrograph, and that of ~50,000 from an ~20,000 resolution linear grating spectrograph.

Subject headings: instrumentation: interferometers — instrumentation: spectrographs — techniques: interferometric

1. INTRODUCTION

An externally dispersed interferometer (EDI) is a spectrograph hybrid of purely dispersive and interferometric systems recently developed for the precision measurement of Doppler radial velocities (Erskine & Ge 2000; Erskine 2002, 2003; Ge, Erskine, & Rushford 2002; Ge 2002). We describe a new application of the EDI: boosting the effective resolving power of a spectrograph by a factor of 2–3 over its full bandwidth. We have demonstrated resolution gain by adding an interferometer to both a linear grating spectrograph and an echelle spectrograph. EDI spectroscopy uses the same apparatus and raw data as used previously for EDI radial velocimetry measurements but applies a different data reduction procedure.

In EDI, fringes created by an external, unequal-arm white-light interferometer are imaged to a spectrograph’s slit. The interferometer produces a transmission comb, sinusoidal in frequency (ν = 1/λ), over the entire bandwidth. This is analogous to the fiducial lines of an absorption cell, but with lines of exceedingly uniform spacing, shape, and amplitude over the entire bandwidth. The comb, in multiplexing with the input spectrum, heterodynes fine spectral features into a low spatial frequency moiré pattern. The heterodyning is numerically reversed to recover highly detailed spectral information otherwise unattainable due to the spectrograph’s resolution or detector Nyquist limits.

The EDI differs from previous hybrids (the Holographic Heterodyning Spectrograph and the Spatial Heterodyning Spectrograph) by using a dispersing element external to the interferometer rather than internal to it (Douglas 1997; Frandsen, Douglas, & Butcher 1993; Harlander, Reynolds, & Roesler 1992). Thus, the resulting EDI fringe spectral periodicity is uniform at all frequencies, allowing for a large operating bandwidth. In comparison with Fabry-Perot interferometer hybrids producing narrow spectral fiducials (Born & Wolf 1980, pp. 333–338; McMillan et al. 1993), the EDI’s sinusoidal fiducial fringes transmit a greater average flux and offer a simple Fou-
input spectrum \( S_0(\nu) \) and the spectroscope line-spread function \( \text{LSF}(\nu) \),

\[
B_{\text{ord}}(\nu) = S_0(\nu) \otimes \text{LSF}(\nu).
\] (1)

The FWHM of \( \text{LSF}(\nu) \) is \( \Delta \nu \) and is related to the resolution by \( R = \nu \Delta \nu \). The convolution of equation (1) expressed in Fourier space is

\[
b_{\text{ord}}(\rho) = s_0(\rho) \text{lsf}(\rho),
\] (2)

where the lower case symbols are the transformed versions and \( \rho \) is the spatial frequency along the dispersion axis in features per cm\(^{-1}\). The \text{lsf}(\rho) is thus the transfer function of the impulse response \( \text{LSF}(\nu) \). The normalized interferometer transmission \( T'(\nu) \) is a sinusoidal spectral comb,

\[
T'(\nu) = 1 + \gamma \cos(2\pi \nu \tau + \phi),
\] (3)

where \( \gamma \) is the interferometer visibility, assumed to be unity for now, and \( \tau \) and \( \nu \) are in units of cm and cm\(^{-1}\), respectively. The spectra \( B_n \) are recorded at multiple phases \( \phi \) differing by \( \sim 90^\circ \), designated \( B_0, B_{90}, \text{ etc.} \). (As few as three \( B_n, \sim 120^\circ \) apart, may be used.) Transmission through the interferometer multiplies \( T'(\nu) \) with the spectrum \textit{prior} to blurring by the external spectrograph. Hence, the EDI-detected signal is

\[
B_{\text{ord}}(\nu) = [S_0(\nu)T'(\nu)] \otimes \text{LSF}(\nu).
\] (4)

Equation (4) is reexpressed as a sum of the ordinary spectrum plus two complex counterrotating fringing terms

\[
B_{\text{ord}}(\nu) = B_{\text{ord}}(\nu) + \frac{1}{2}[S_0(\nu)e^{i\nu}\nu e^{-i\nu} + S_0(\nu)e^{-i\nu}e^{i\nu} \nu] \otimes \text{LSF}(\nu).
\] (5)

High-resolution information is recovered by isolating a fringing component and reversing the heterodyning. The scalar spectrum is converted to a complex vector spectrum called a "whirl," \( W(\nu) \), by using a linear combination of the phased exposures \( B_n \) that have been numerically rotated in synchrony with each exposure's phase-step value. The whirl for four-phase recording every 90° is

\[
W(\nu) = \frac{1}{4}(B_0e^{i\nu} + B_{90}e^{i\nu} + B_{180}e^{-i\nu} + B_{270}e^{-i\nu})
\] (6)
or

\[
W(\nu) = \frac{1}{4}[(B_0 - B_{180}) + i(B_{90} - B_{270})].
\] (7)

Applying equation (6) or equation (7) to equation (5), we get

\[
W(\nu) = \frac{1}{4}[e^{i\nu}S_0(\nu)] \otimes \text{LSF}(\nu).
\] (8)

The Fourier transform of the whirl is then

\[
w(\rho) = \frac{1}{4} \gamma s_0(\rho + \tau) \text{lsf} (\rho),
\] (9)

where we include the interferometer visibility \( \gamma \) previously taken as unity. This important equation describes the EDI formation of moiré fringes, a heterodyning effect expressed in the \( s_0(\rho + \tau) \) argument. Fine spectral details having high \( \rho \) are heterodyned (shifted by \( \tau \)) to low \( \rho \), prior to blurring by the spectrograph’s line-spread function.

To obtain the EDI spectra, the optical heterodyning is numerically reversed, and the ordinary and fringing spectral components are combined. The latter is analogous to combining the “treble” and “bass” channels of an audio system to obtain a more full-bodied sound. The bass channel is the ordinary spectrum, and the treble channel is the fringing spectrum containing higher resolution information. The ordinary spectrum is determined by summing the phase-stepped data so that the fringing terms are canceled out,

\[
B_{\text{ord}}(\nu) = \frac{1}{4}(B_0 + B_{180} + B_{90} + B_{270}).
\] (10)
The factors $\alpha$ and $\beta$ are chosen to avoid noise from spectral regions with insignificant signals. An optimal signal-to-noise ratio ($S/N$) occurs when $\alpha$ and $\beta$ are proportional to the bass and treble response functions. Finally, the real spectrum $B_{\text{net}}(\nu)$ is obtained from the inverse transform of $b_{\text{net}}(\rho)$.

The inflection in the net spectral response function $b_{\text{net}}(\rho)$ (Fig. 3, thick solid curve) indicates that EDI sensitivity differs with $\rho$. Response equalization, analogous to audio bass and treble equalization, removes the inflection and can shape the final profile [LSF$_{\text{net}}(\nu)$] to a desirable Gaussian and eliminate ringing in the LSF. Calibration spectra establish the scaling for equalization. During equalization, both the signal and the noise per $\rho$ are equally scaled.

4. EXPERIMENTAL DEMONSTRATIONS

Figure 1 demonstrates EDI resolution boosting on solar spectra using an $R = 20,000$ linear grating, $\tau = 1.1$ cm, and the multiphase fringe mode. The measured composite spectra $B_{\text{net}}(\nu)$ compare closely with a very high resolution solar spectra from the Kitt Peak Fourier transform spectrometer (FTS) that has been artificially blurred to $R = 60,000$, demonstrating a resolution boost of $\sim 2.5$ times while using the original $R = 20,000$ slit width.

Figure 2 shows a portion of the EDI raw spectra of $\epsilon$ Leo using the Lick Observatory two-dimensional echelle grating spectrograph (Vogt 1987), a $\tau = 1.1$ cm interferometer, and the uniphase mode. The spectrum was recorded over the full echelle bandwidth ($\sim 4000$–$8000$ Å), and only a tiny subset is shown. The change in the fringing moiré pattern due to a $150^\circ$ phase step shows the presence of information near the $1/\tau = 0.9$ cm$^{-1}$ interferometer periodicity.

EDI uniphase spectra of $\alpha$ Virginis (B1 III) were acquired using the Lick echelle spectrograph and $\tau = 3.3$ cm. Figure 4 shows a small subset of the EDI spectra about the oxygen telluric lines ($\sim 6868$ Å) and Kitt Peak FTS solar reference spectra. The measured EDI ordinary spectrum, with $R = 53,000$ as limited by the echelle spectrograph, cannot resolve several split lines. The split line features are resolved by including the high-frequency features in the EDI fringing spectrum. The $\sim 2$ times EDI effective resolution gain is seen by comparing the reference solar spectrum, blurred to $R = 53,000$ and $R = 96,000$, with the ordinary and EDI composite spectra. The resolution gain occurs over the entire echelle bandwidth. The comparison of the telluric lines between starlight and sunlight is appropriate because both are subject to similar absorption by oxygen in the Earth’s atmosphere.

Figure 3 shows the EDI and conventional Lick echelle instrument responses versus $\rho$. The $R = 50,000$ grating used alone is not sensitive to features with $\rho > 3$ cm. Introducing the interferometer shifts the grating response to $\rho = 3.3$ cm, where the $\rho > 3$ cm response is nearly equal to that of a model Gaussian $R = 100,000$ grating. Hence, for high $\rho$, where the important science information resides, the effective resolution has been boosted $\sim 2$ times. A boost of 2.5 times would theoretically occur for $\gamma = 1$ and $\tau = 3.6$ cm. A standard interfer-
To improve the observed sensitivity, we consider EDI sensitivity and noise, leaving a complete treatment to future publications. The EDI noise per spectral interval $\sigma_{B_{\text{net}}}^2$ in the combined EDI spectra $B_{\text{net}}(\rho)$ is determined by the noise in the ordinary $B_{\text{ord}}$ and fringing $B_{\text{edi}}$ spectral components. The noise for the ordinary component (see eq. [10]) is the same (in both $\rho$- and $\rho'$-space) as that for the conventional grating spectrograph for the same total exposure time because each phase-recording $B_{\text{r}}$ is an independent measurement.

The noise for the EDI and ordinary components are similar because the EDI component (eq. [6]) is constructed of the same $B_{\text{r}}$ terms as the ordinary component (eq. [10]), differing only by unity magnitude phasors. The phasors do not affect the net noise statistically because $B_{\text{r}}$ are independent and because the phasors only rotate $B_{\text{r}}$. Random noise in $B(\rho)$ produces uniform random noise in $b(\rho)$. Hence, the same noise magnitude per $\rho$-bin exists for both $b_{\text{ord}}(\rho)$ and $b_{\text{edi}}(\rho)$. Furthermore, the noise per $\rho$-bin in $b_{\text{ord}}(\rho)$ is almost the same as that for its components $b_{\text{ord}}$ and $b_{\text{edi}}$ because the net response is formed from a weighted composite (eq. [11]) that effectively includes only one of the two components at each $\rho$. Hence, before equalization,

$$
\sigma_{B_{\text{net}}}^2 \approx \sigma_{B_{\text{ord}}}^2 \approx \sigma_{B_{\text{edi}}}^2.
$$

Equalization will affect noise in the final spectrum. The equalization process scales the noise per $\rho$-bin for only the mid-$\rho$ region. Therefore, noise for high spectral resolution features (high $\rho$) is not affected. Noise for mid-$\rho$ features will be scaled by a factor $U$, the ratio of equalized to unequalized responses (see Fig. 3) at the mid-$\rho$ response notch. The value of $U$ will depend on $r/R$ and $\gamma$ and is ~2 here. For broad ($\rho < 1$ cm), midrange ($\rho \sim 2$ cm), and narrow ($\rho > 3$ cm) spectral feature sizes, for example, our EDI instrument’s predicted net continuum-to-noise ratios (C/Ns) are ~80%, ~50%, and ~100% of the conventional C/N of an $R = 100,000$ grating, respectively. The C/N for the unequalized EDI net and conventional spectra is ~37 at 18 pixel Å$^{-1}$, or $\sigma_{B_{\text{ord}}^2} \sim \sigma_{B_{\text{net}}^2}$.

3%. Our equalized net EDI spectrum (Fig. 4) noise is $\sigma_{B_{\text{net}}}^2 \sim 5\%$, or $\text{C/N} \sim 20$. This is consistent with the upper limit $\sigma_{B_{\text{net}}}^2 \leq U\sigma_{B_{\text{net}}}^2$ found when the noise at all $\rho$ is scaled by $U$.  

4.2. Multiple-Delay EDI

Relative to grating spectrographs alone, the EDI is several times more compact per resolution and provides a more stable LSF for high spectral details. Further development of the EDI promises benefits to multiobject and slit imaging spectroscopy and to smaller and more economical high-resolution instruments. Versions of EDI having multiple delays can achieve higher resolution with a modest trade-off of photon noise. The EDI interferometer can be modified to simultaneously imprint a delay $\tau_{\text{m}}$ on $M$ different paths through the same spectrograph and to detect these on separate regions of the detector. This forms a multiple-delay EDI that, if $\tau_{\text{m}}$ discretely covers $\rho$-space, yields a spectrograph with resolution boosting roughly proportional to $2M$. The photon S/N will decrease approximately proportional to $1/\sqrt{M}$ because the total flux is subdivided into $M$ channels. For example, an $R = 20,000$ grating using a three-delay EDI having $\tau_{\text{m}} = 1, 2$, and 3 cm can attain a resolution boost of ~7 times. As $M \rightarrow \infty$, we approach the regime of an FTS that scans $\tau$ quasi-continuously in $M$ exposures. In Figure 3, the FTS response function would have $M$ very narrow peaks of $1/\sqrt{M}$ height distributed over the $\rho$-range of interest.

Hence, the single-delay EDI photon S/N is better than that of the FTS by ~$\sqrt{M} \sim [R(BW/\lambda)]^{1/2} \sim 100$ for an $R = 20,000$, a bandwidth $BW = 4000$ Å, and average $\lambda = 6000$ Å (e.g., Beer 1992, p. 66).

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